Criticalidade auto-organizada

(self-organized criticality)

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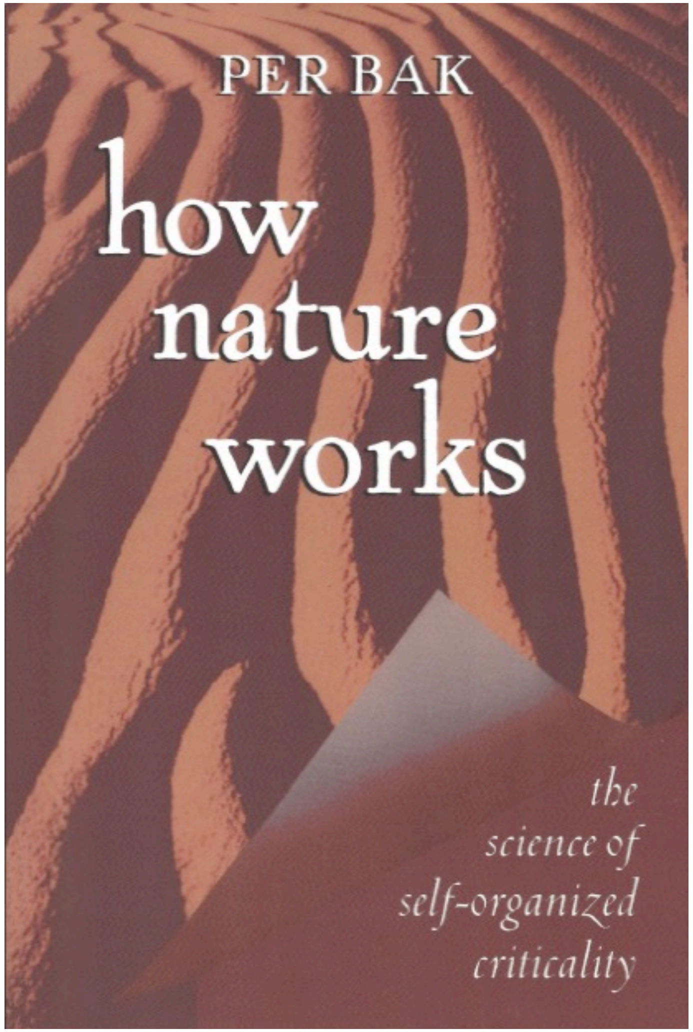
http://www.namaraujo.net

Bibliography



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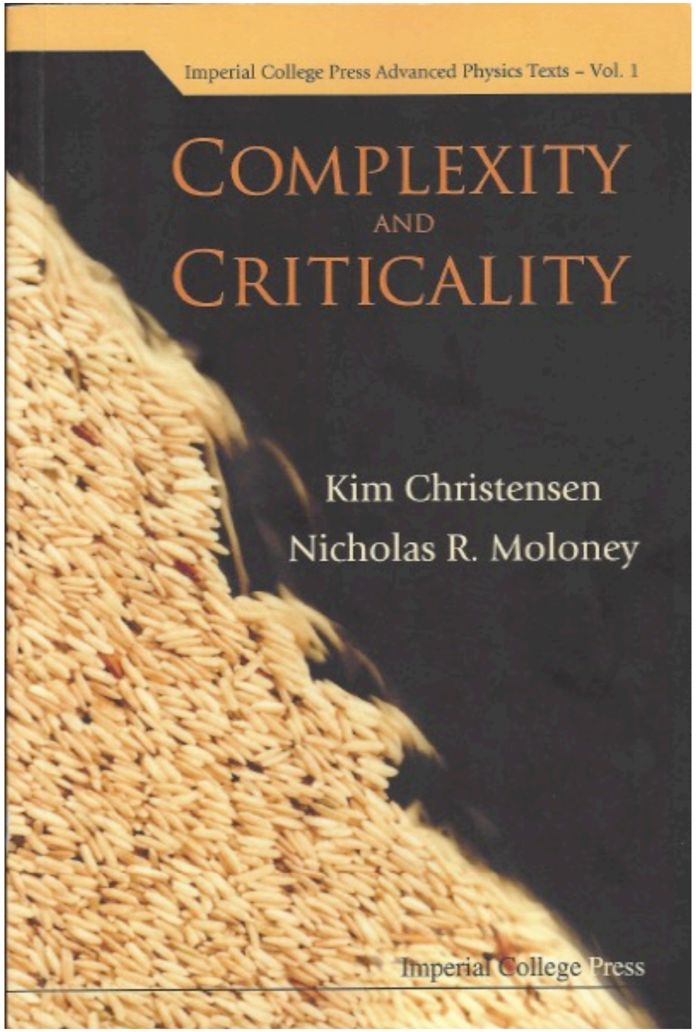

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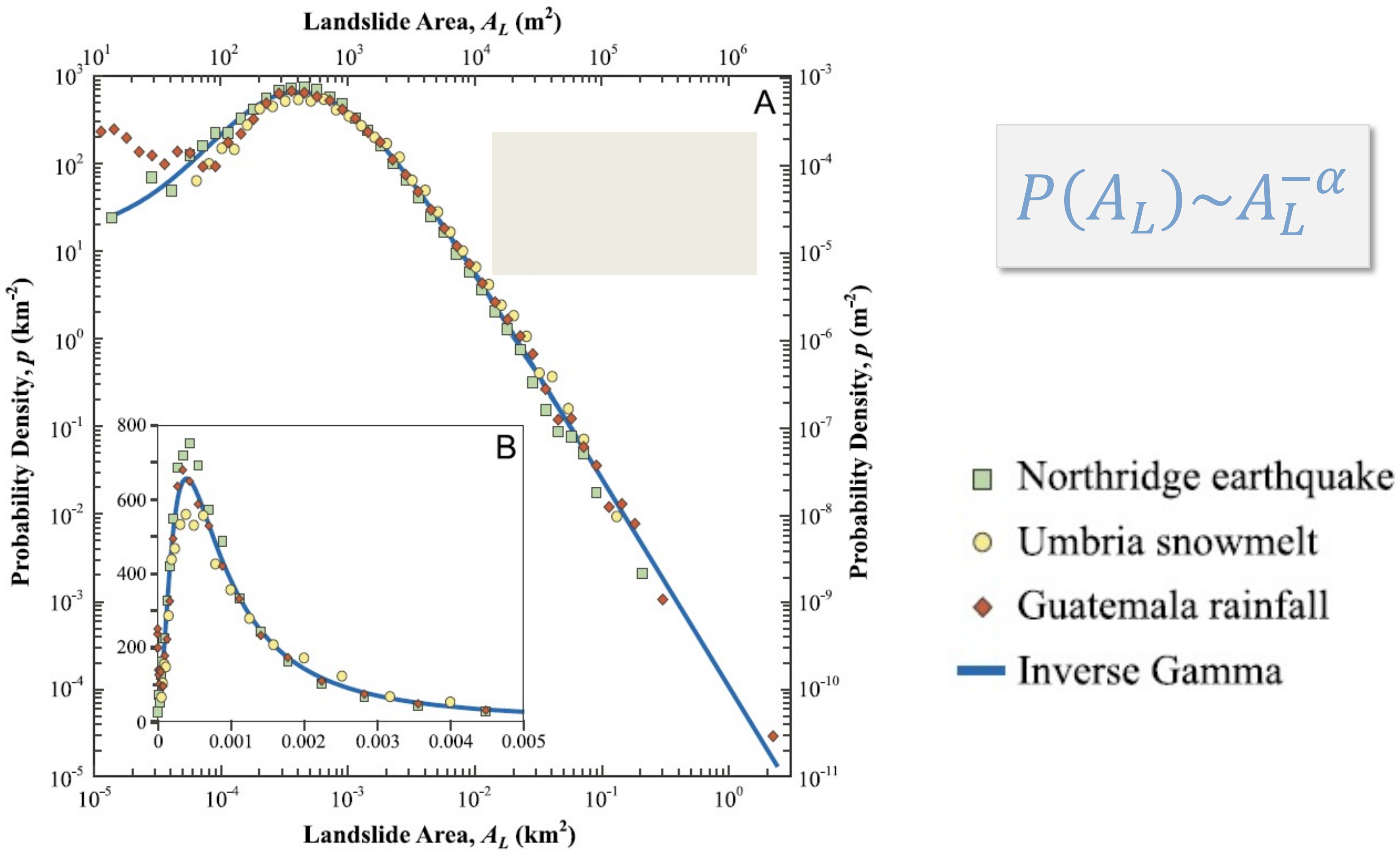
Landslides size distribution



$$P(A_L) \sim A_L^{-\alpha}$$

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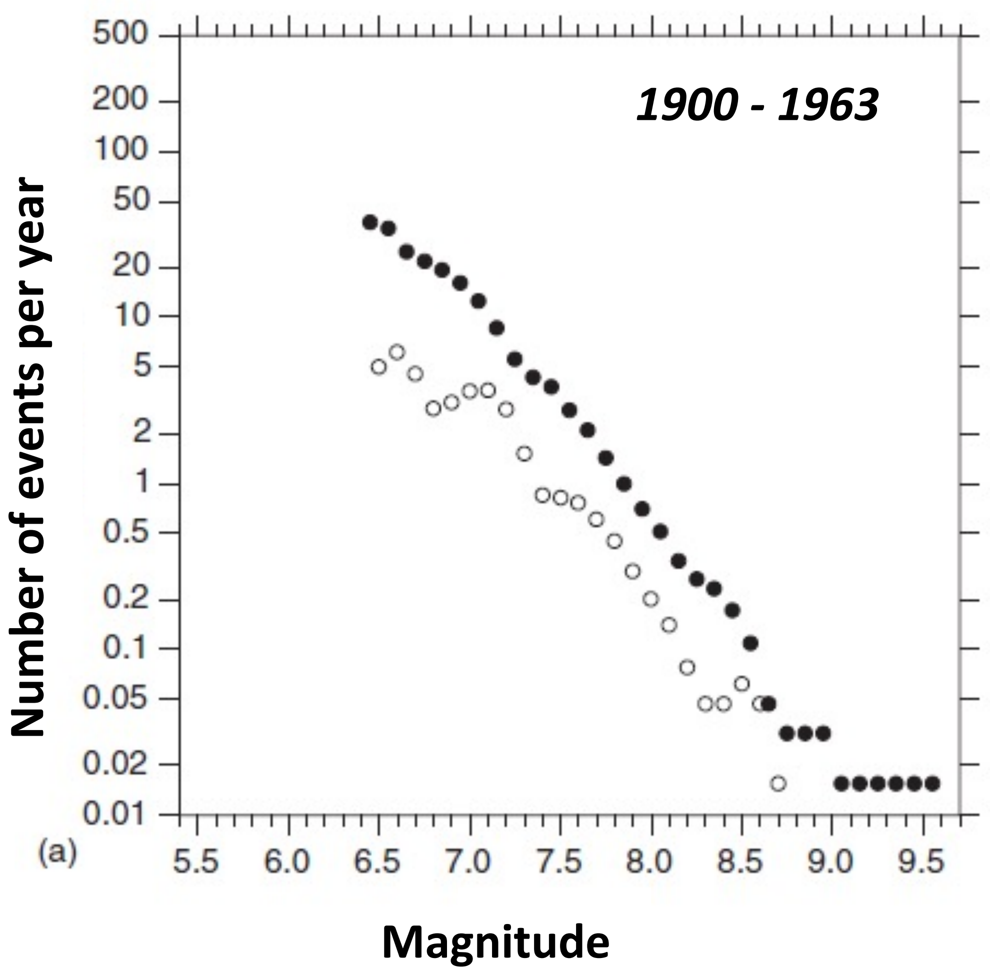
Earthquakes
Gutenberg-Richter law

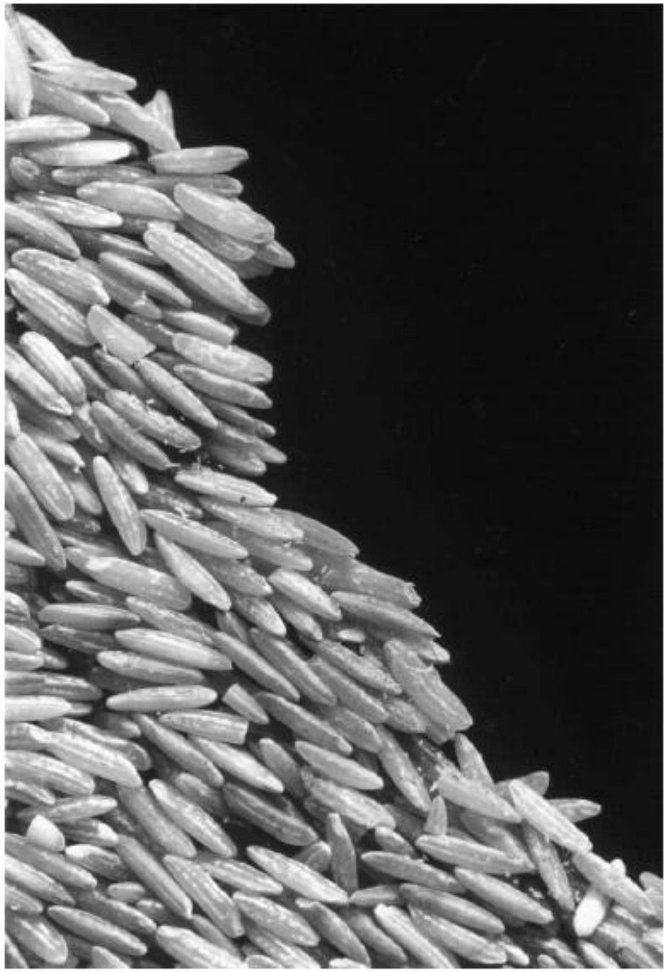


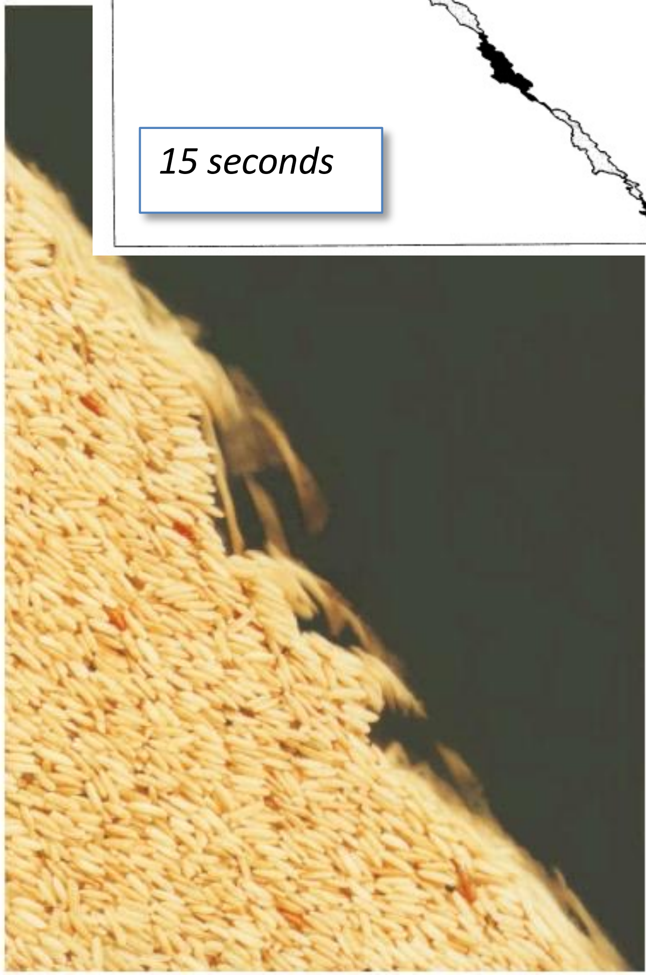
$$N(m) \sim m^{-\alpha}$$

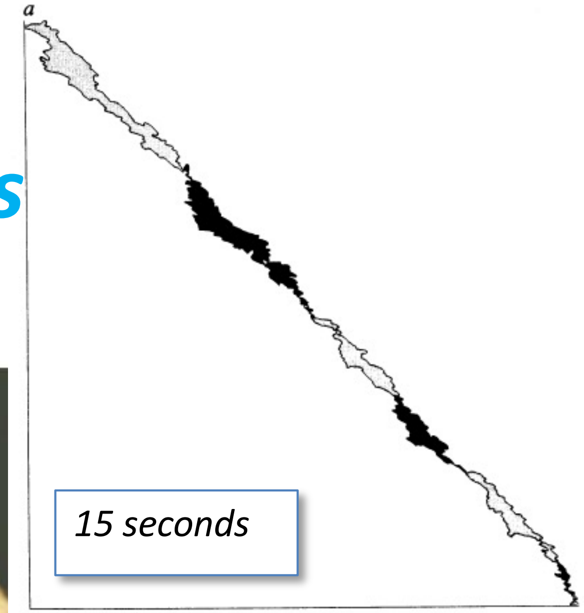
m: Magnitude

http://earthquake.usgs.gov/research/data/centennial.pdf

Avalanches
Norwegian rice piles







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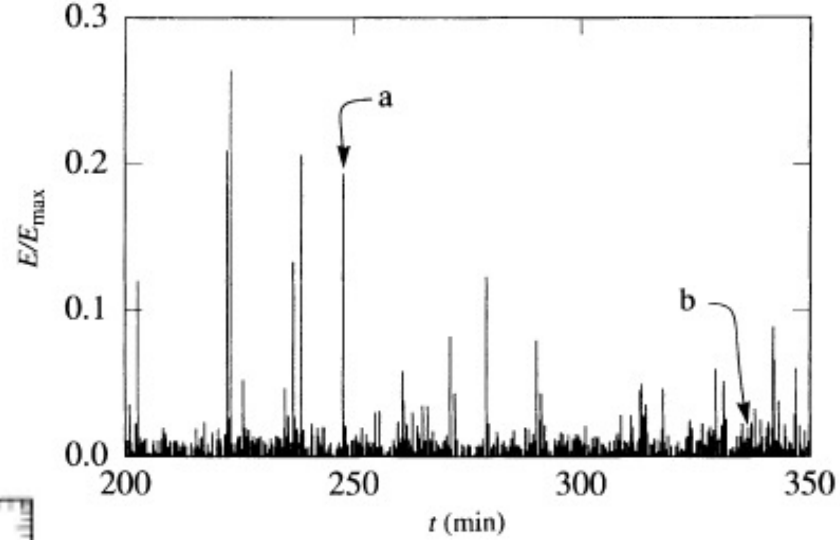
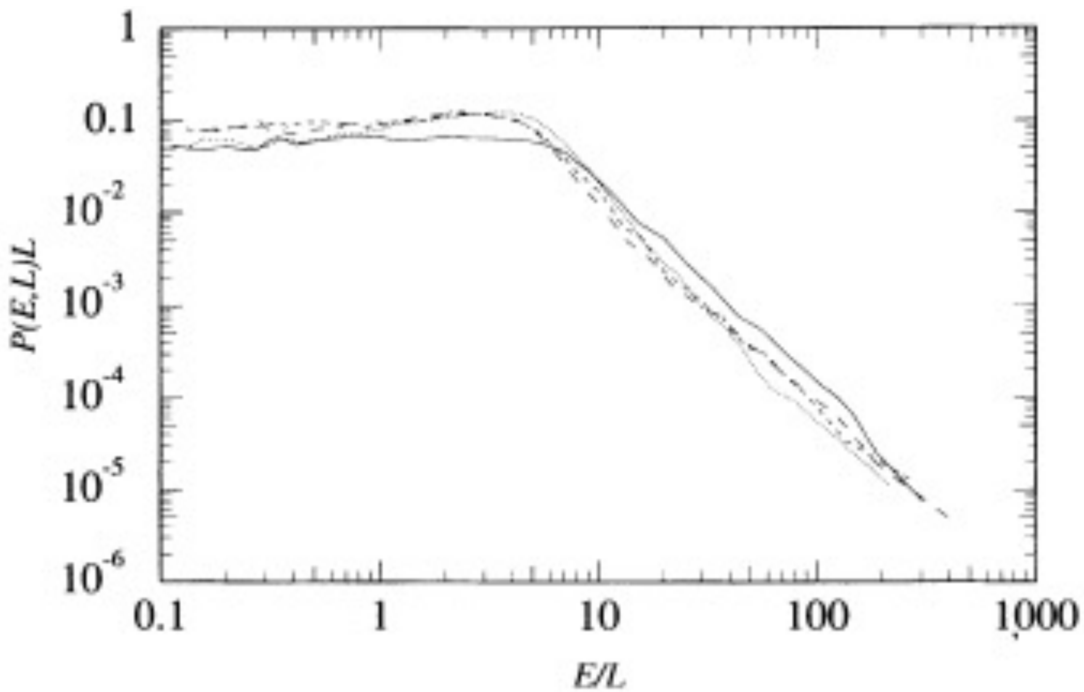
Avalanches

Norwegian rice piles





$$P(E) \sim E^{-\alpha}$$

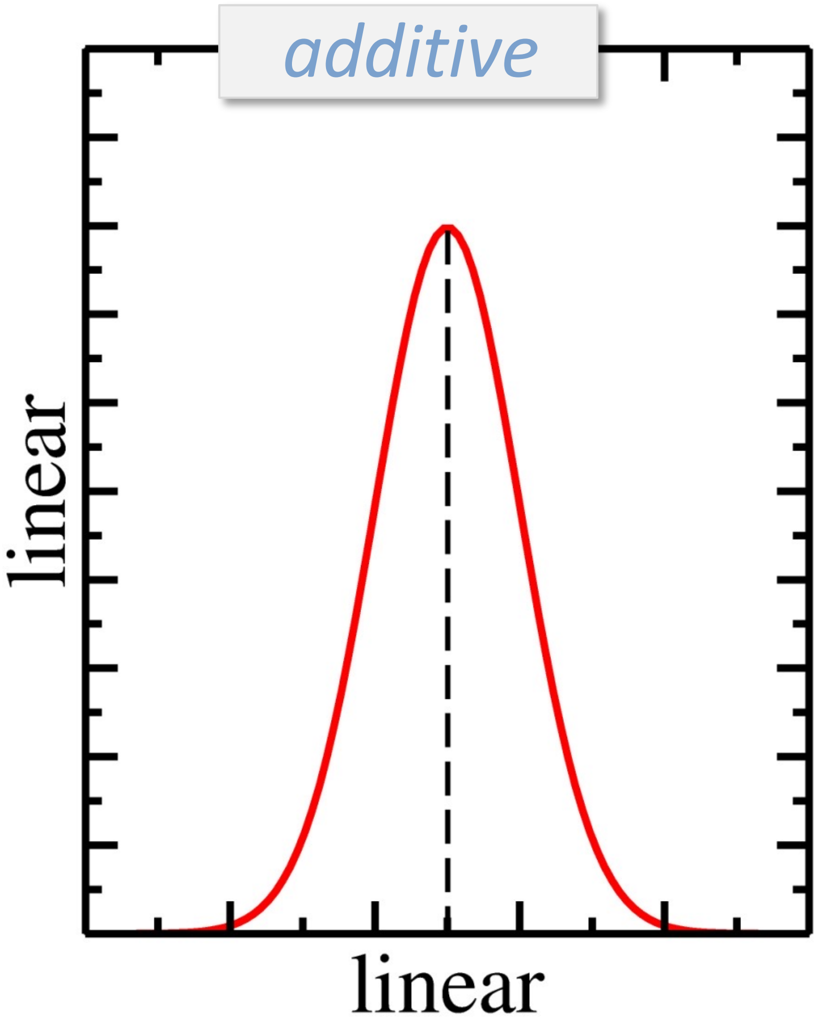
E: Avalanche size

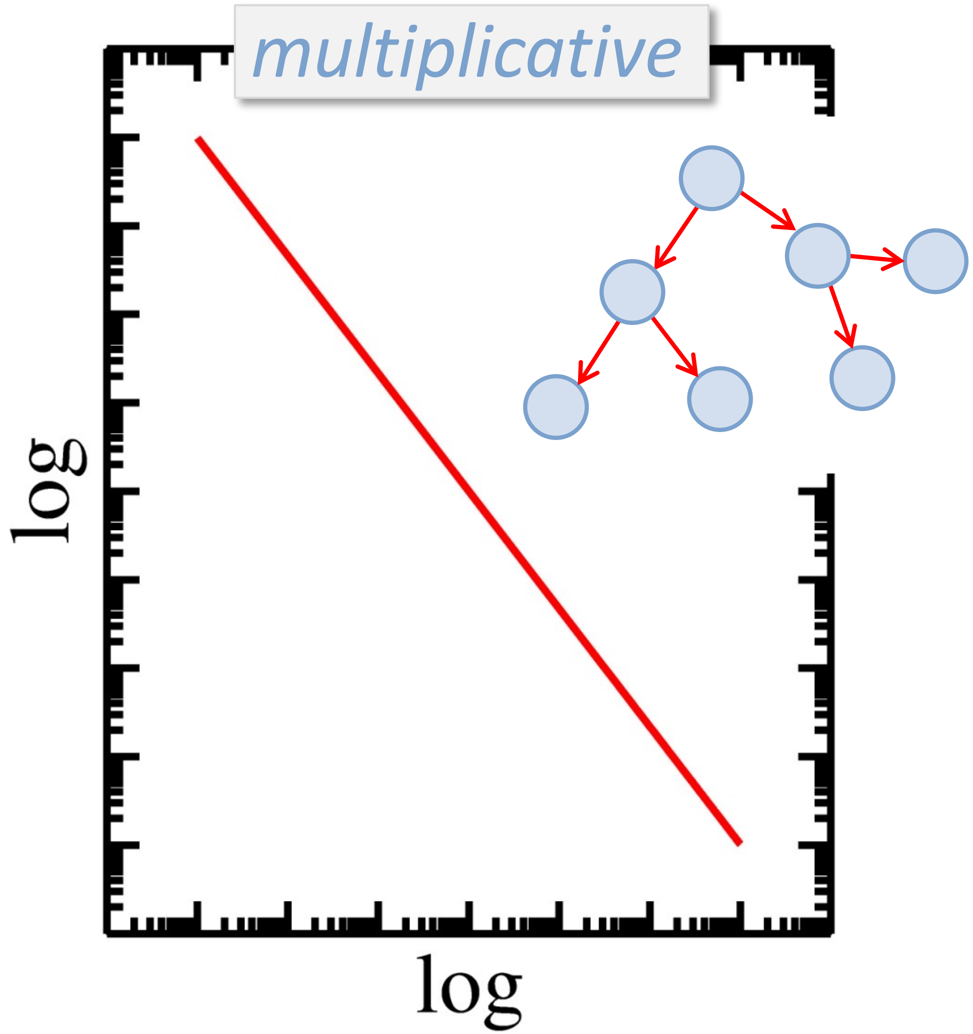
L: Pile size

V. Frette *et al*. *Nature* **379**, 49 (1996)

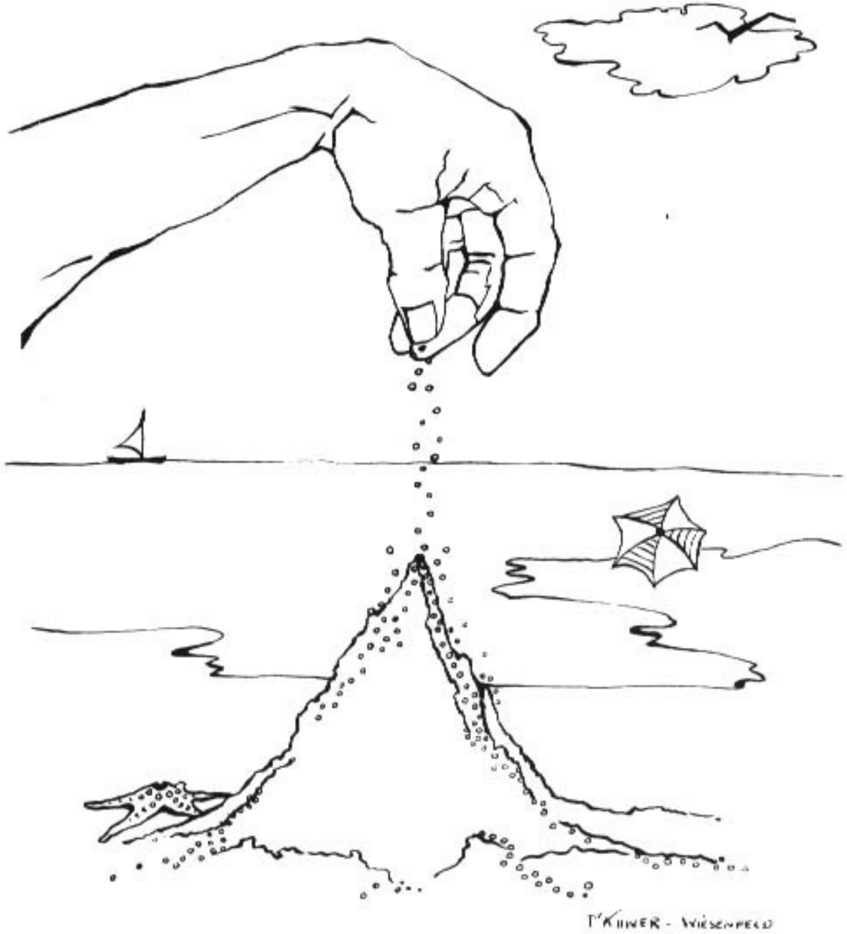
Additive vs. multiplicative processes

Gaussian vs. Power law





Sandpile model
Bak-Tang-Wiesenfeld (BTW) model



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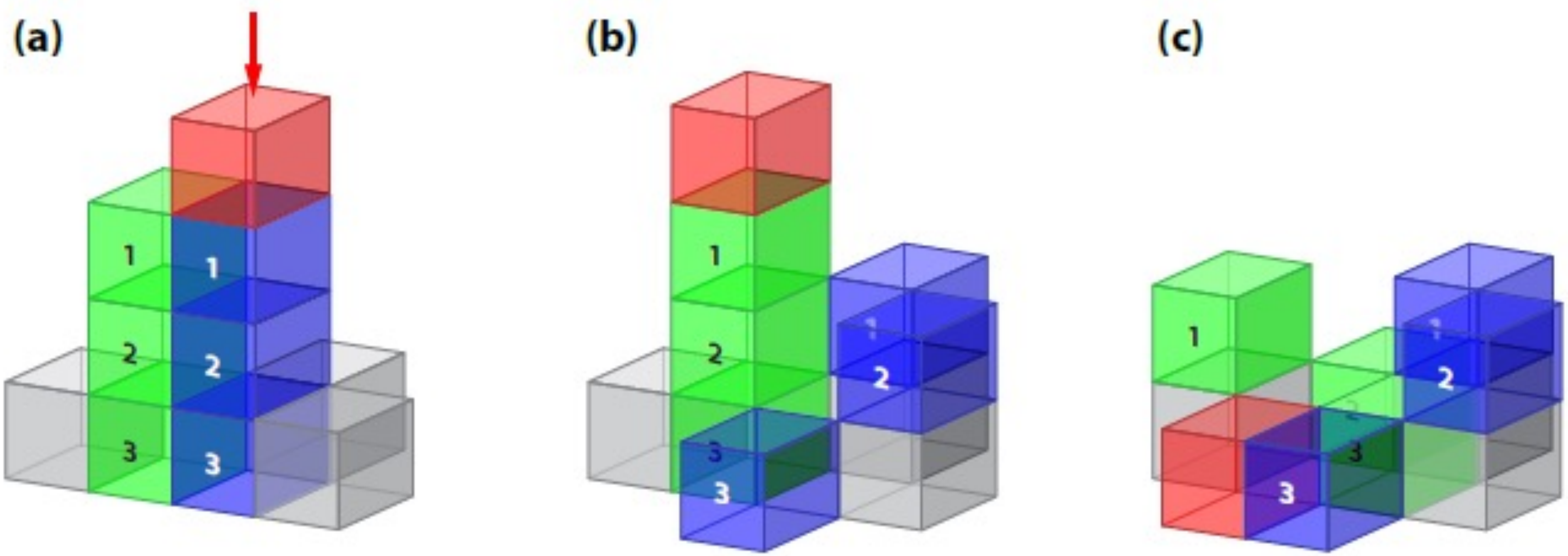
Sandpile model
BTW model

- slowly ***driven***;
- ***store*** mass (or energy);
- local ***threshold***;
- ***relax*** intermittently.



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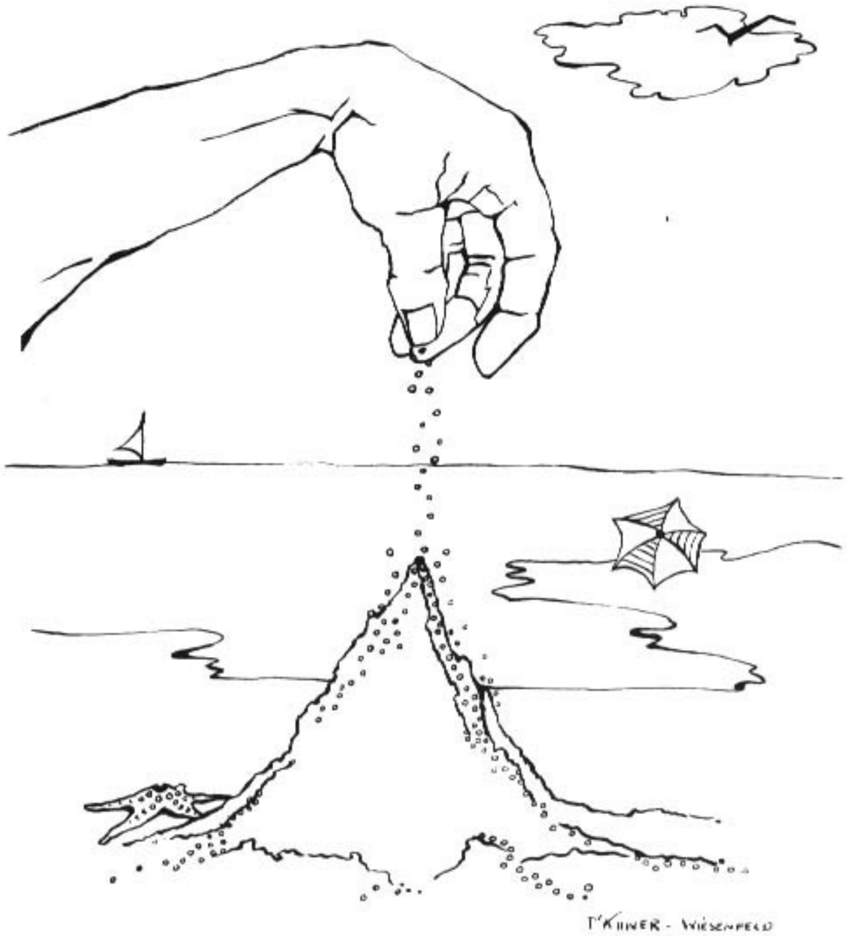
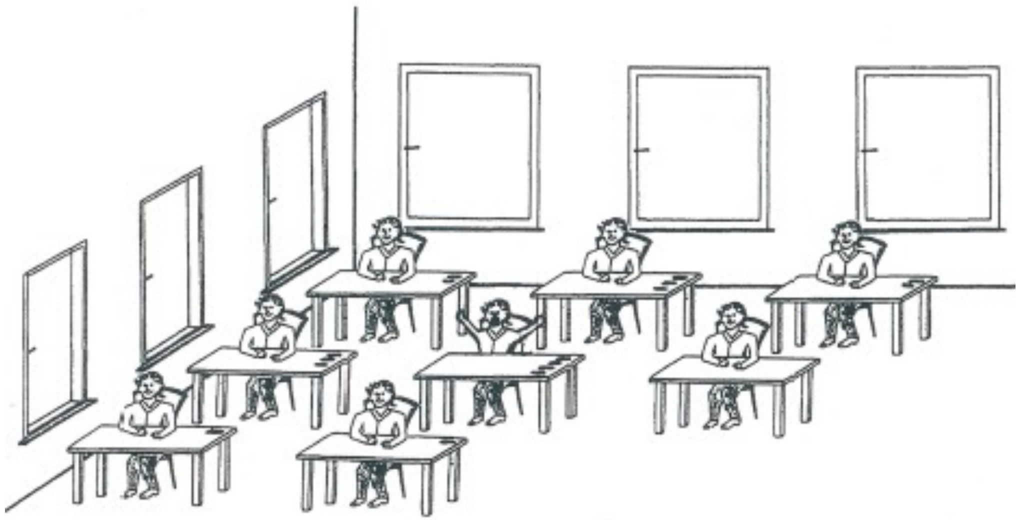
Sandpile model
Bak-Tang-Wiesenfeld (BTW) model





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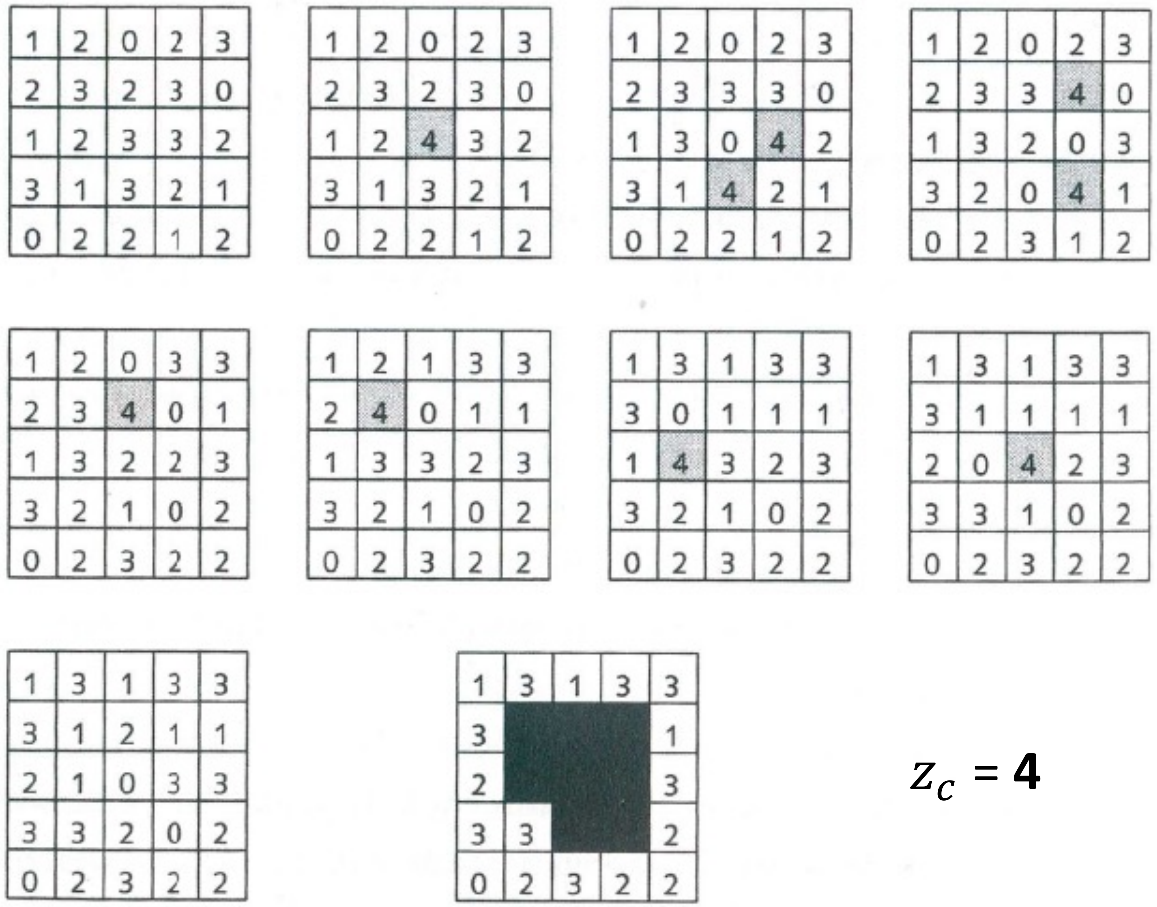
Sandpile model
BTW model

$z(x,y) \rightarrow z(x,y) + 1$

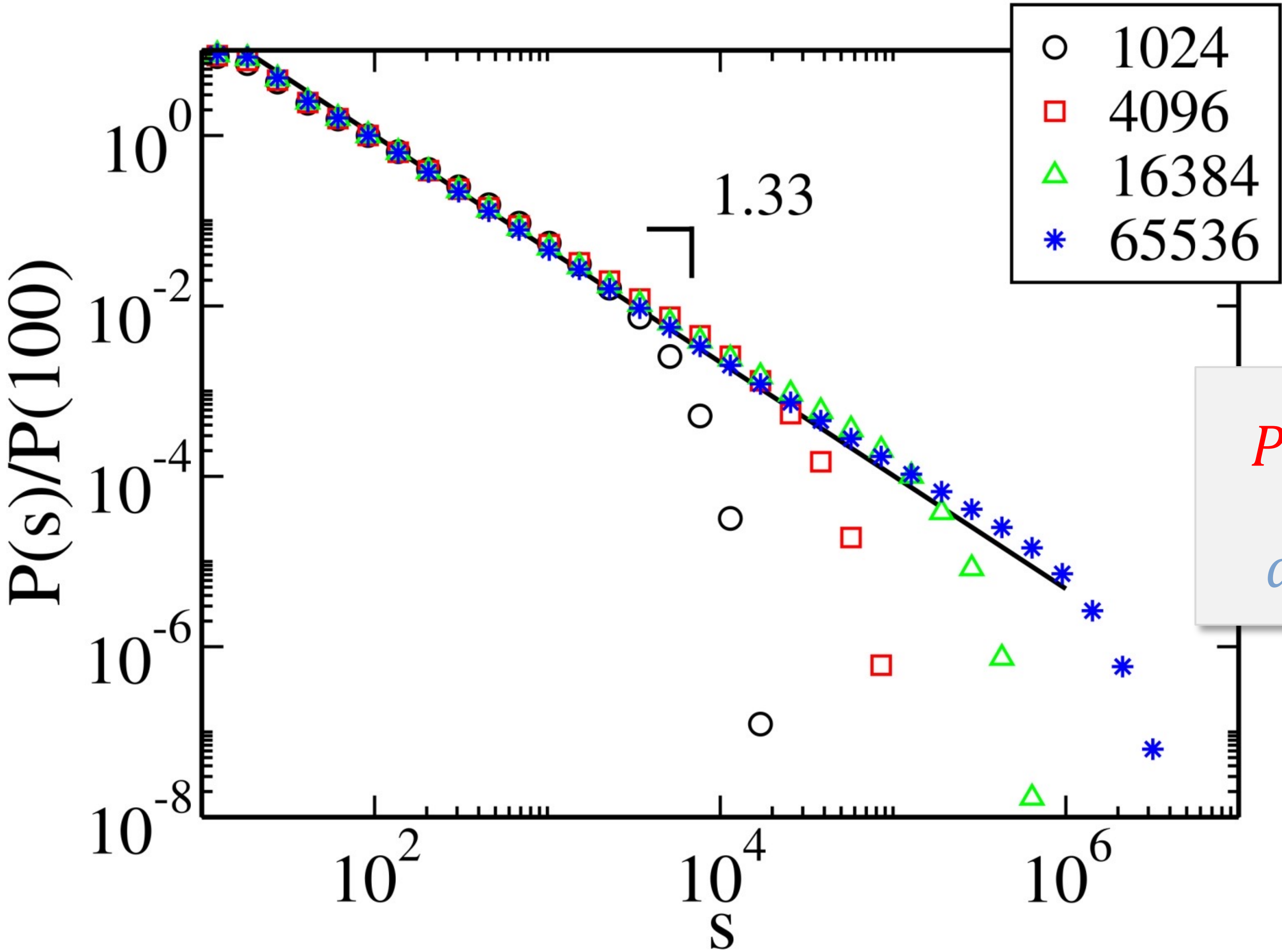
if $z(x,y) > z_c$ ***then***
avalanche



z_c = **4**

P. Bak, C. Tang, and K. Wiesenfeld. *Phys. Rev. Lett.* **59**, 381 (1987)
P. Bak. *How nature works: the science of self-organized criticality*. Springer-Verlag, New York, USA (1996)

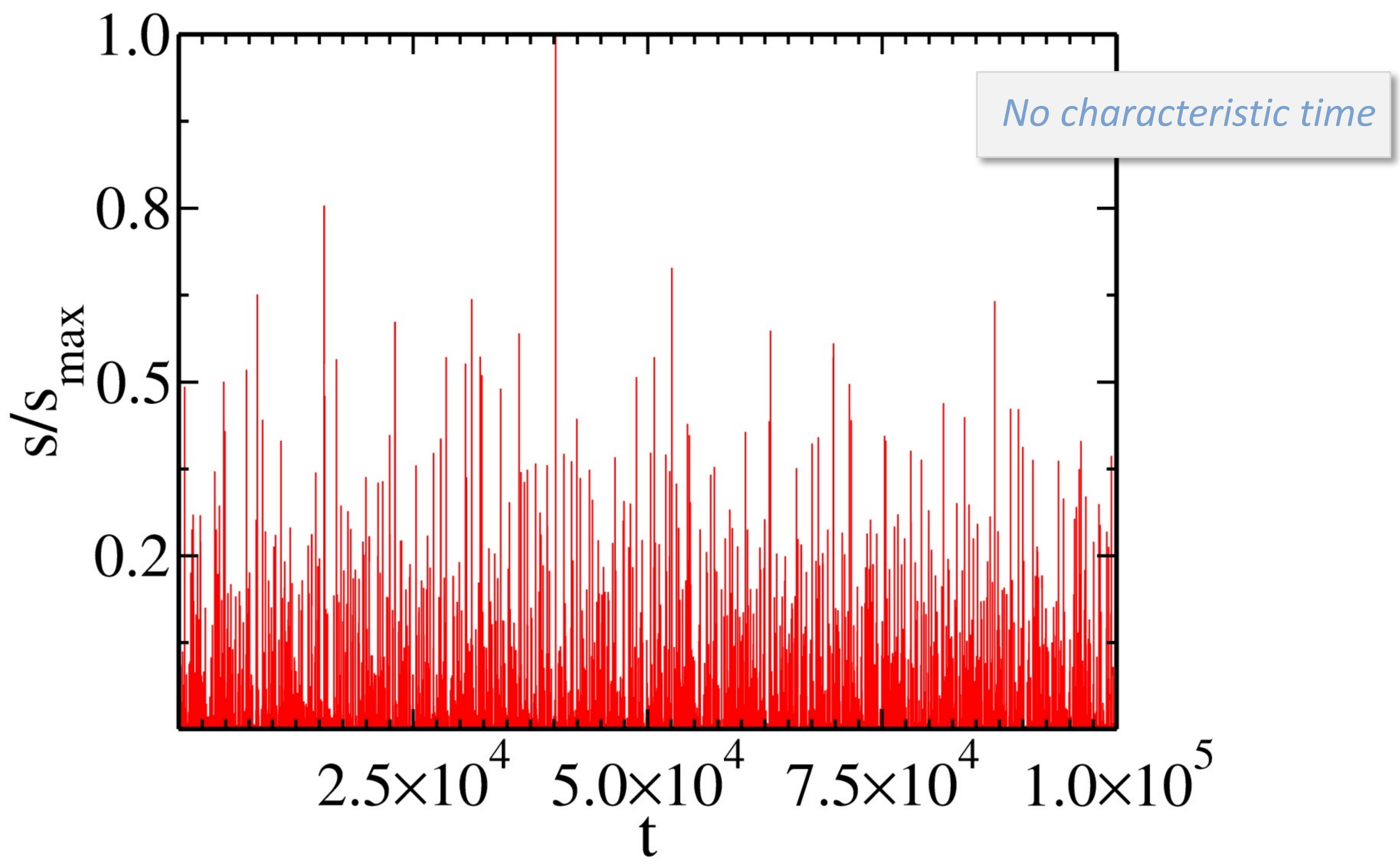
BTW model
Avalanche-size distribution



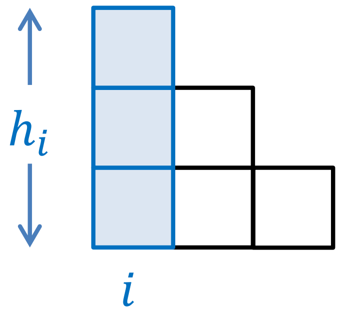
$P(s) \sim s^{-\alpha}$

$\alpha \approx 1.33$

BTW
Distribution of waiting time



BTW model
1D



Boundary conditions:
- *sealed* at the left;
- *open* at the right.

$i = 1, 2, \dots, L$

$z_i = h_i - h_{i+1}$, and $h_{L+1} = 0$

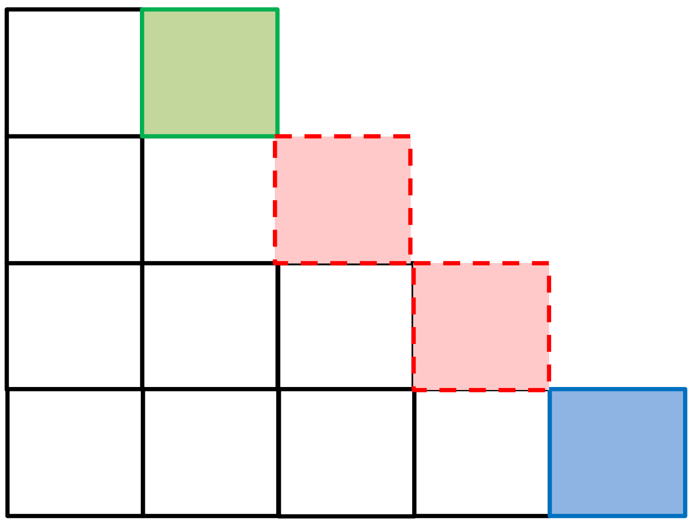
Threshold: $z_c^i \equiv z_c = 1$

Drive: $h_i \to h_i + 1$

if $z_i > z_c$ ***then*** avalanche

$h_i \to h_i - 1$
$h_{i+1} \to h_{i+1} + 1$

Repeat until all $z_i < z_c$



BTW model
Stable configurations

$$\mathcal{T}_0 \to \mathcal{T}_1 \to \cdots \to \mathcal{T}_{n-1} \to \mathcal{R}_n \to \mathcal{R}_{n+1} \to \cdots$$

Transient

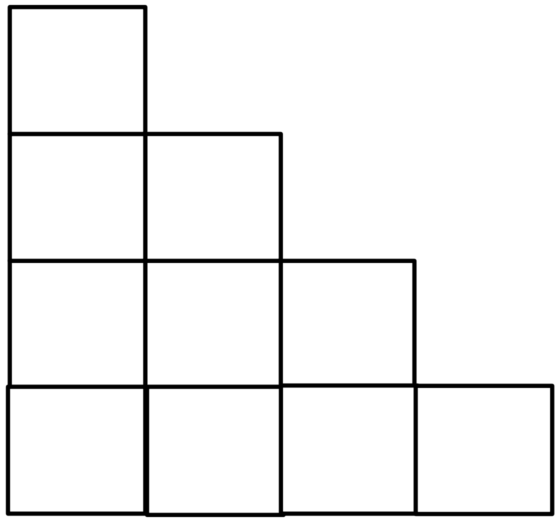
... are not found when the system has entered the set $\mathcal{R}$.

Recurrent

... are revisted indefinitely.

BTW model
Stable configurations (1D)

There is only one recurrent configuration



$$\langle influx \rangle = \langle outflux \rangle$$

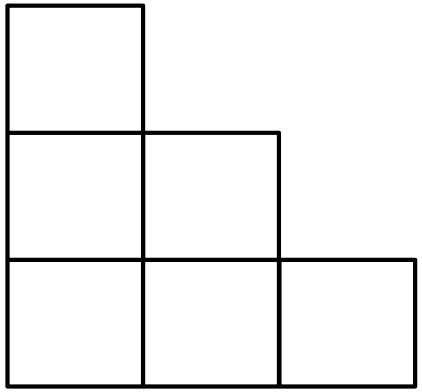
BTW model
Avalanche size distribution (1D)

$$s_{max} = L$$

$$1 \leq s \leq L$$

$$s = L + 1 - i$$

$$\left.\begin{matrix} s_{max} = L \\ 1 \leq s \leq L \\ s = L+1-i \end{matrix}\right\} \langle s \rangle = \frac{L+1}{2}$$



$$P(s,L) = \begin{cases} \dfrac{1}{L}, & 1 \leq s \leq L \\ 0, & otherwise \end{cases}$$

BTW model
mean field

$i = 1, 2, \dots, N$

Related to dissipation

Threshold: $h_c^i \equiv h_c = n/\alpha_n$

Drive: $h_i \to h_i + 1$

if $h_i > h_c$ ***then*** avalanche

$h_i \to h_i - h_c$
$h_j \to h_j + 1, \text{for } n \text{ sites}$

Repeat until all $h_i < h_c$

There are $\boldsymbol{N_s = (h_c)^N}$ stable configurations.

Probability $P(h)$ that a site contains h units:

$$P(h) = \frac{1}{h_c} \quad h = 1, \dots, h_c$$

$$\langle h \rangle = \sum_{h=1}^{h=h_c} h\, P(h) = \frac{h_c + 1}{2}$$

The probability of initiating an avalanche: $P(h_c)$

$$\langle influx \rangle = \frac{1}{P(h_c)} = h_c$$

$$\langle outflux \rangle = (1 - \alpha_n) h_c \langle s \rangle$$

↓

$$\langle s \rangle = \frac{1}{(1 - \alpha_n)}$$