Single-Dish Radio Observations

- We wish to measure the flux density F_ν of a source. This is related to its specific intensity: $F_\nu \sim I_\nu \, \Omega$.
- The intensity can be expressed in terms of *brightness temperature*. In the R-J regime:

$$I_\nu (\theta, \phi) = \frac{2k}{\lambda^2} T_B (\theta, \phi)$$

- The power of the radiation collected from a source with flux density F_ν is: $P = F_\nu A_{eff} \, \Delta\nu$

  ($\Delta\nu$ = bandwidth or range of frequencies detected)

  Power from the astronomical source: $P = k \, \Delta\nu \, T_A$
- So, in principle, if we measure T_A we get F_ν. So it seems simple. But it is complicated!

Single-Dish Radio Observations

- First, let's consider the noise.
- Power from the astronomical source: T_A
- Power from the receiver: T_N (or T_{Rec})
- All un-wanted extra power: $T_{\rm sys} = T_N + T_{\rm atm} + \ldots$
- The *system temperature* $T_{\rm sys}$ includes power from the:
  - receiver
  - background sky
  - atmosphere
  - reflector
  - spillover

Single-Dish Radio Observations

- Power from the astronomical source: T_A
- All un-wanted extra power: T_{sys}
- Amplifiers: gain G
- Total power: $P = G\, k\, \Delta\nu\, (T_A + T_{\mathrm{sys}})$
- The power measured by the receiver is converted into a voltage: $V_{\mathrm{on}} = \alpha\, G\, k\, \Delta\nu\, (T_A + T_{\mathrm{sys}})$
- But $T_A << T_{\mathrm{sys}} \Rightarrow$ need switching: measure another signal, with same overall power, that differs from the first, solely by the fact that it does not contain the astronomical signal, and subtract.

Switching

- On-source and off-source:
  - $V_{\text{on}} = \alpha\, G\, k\, \Delta\nu\, (T_A + T_{\text{sys}})$
  - $V_{\text{off}} = \alpha\, G\, k\, \Delta\nu\, T_{\text{sys}}$
- $V_{on} - V_{\text{off}} = \alpha\, G\, k\, \Delta\nu\, T_A$
- Switching allows to determine T_A but has to be done as fast as any power variations due to the receiver or the atmosphere.
- Position switching and beam switching:
  - Position switching: the OFF position is specified
  - Beam switching: specify the “throw” (in azimuth or elevation)

Calibration

- $V_{on} - V_{off} = \alpha\, G\, k\, \Delta\nu\, T_A$
- $V_{off} = \alpha\, G\, k\, \Delta\nu\, T_{sys}$
- It seems that we need to measure α and G separately and for every observation. That is not practical.

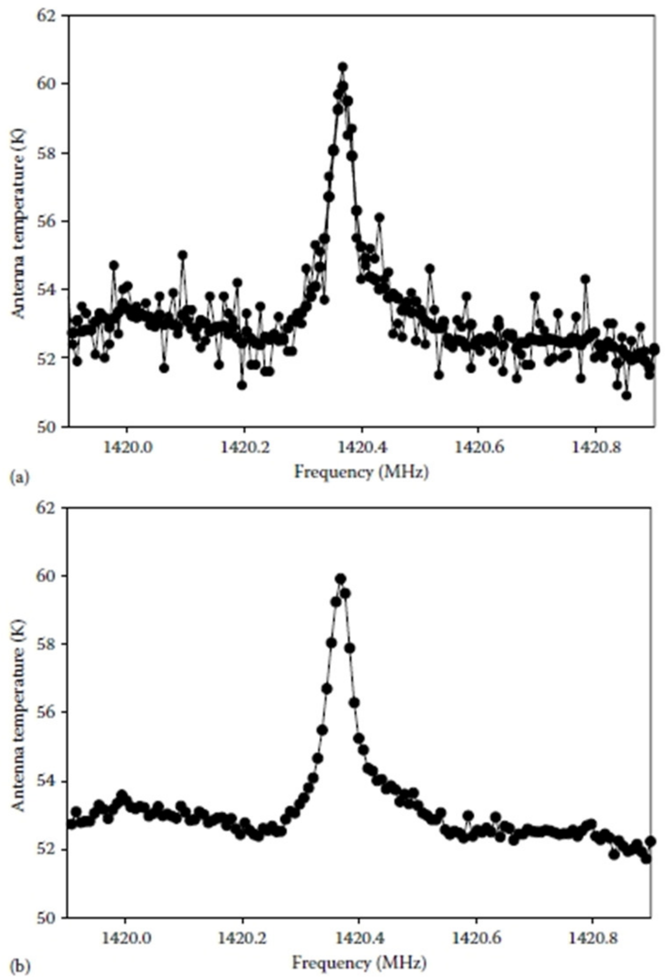
- Calibration of the system: A noise diode injects a known amount of radio-frequency power per unit bandwidth corresponding to a known equivalent temperature T_{cal}.

Measuring the system temperature

- Telescope looking at blank sky:
  - With diode on: V_{cal}
  - With diode off: V_{off}
- $\dfrac{V_{\text{cal}}}{V_{\text{off}}} = \dfrac{T_{\text{cal}} + T_{\text{sys}}}{T_{\text{sys}}} \quad \Rightarrow \quad T_{\text{sys}} = \dfrac{V_{\text{off}}}{V_{\text{cal}} - V_{\text{off}}} T_{\text{cal}}$
- The conversion from voltage to temperature is given by the factor $\dfrac{T_{\text{cal}}}{V_{\text{cal}} - V_{\text{off}}}$.
- We don't need to calculate α and G separately.

Measuring the antenna temperature

- Instead of the diode, we inject the astronomical signal:
  - $\frac{V_{on}}{V_{off}} = \frac{T_A + T_{sys}}{T_{sys}} \Rightarrow$
  - $T_A = \frac{V_{on} - V_{off}}{V_{off}} T_{sys}$
- Uncertainty in T_A:
  - $T_A << T_{sys} \Rightarrow$
  - $\sigma(T_A) = \frac{T_{sys}}{\sqrt{\Delta t \Delta \nu}}$



Uncertainty in T_A: radiometer equation

- $\sigma(T_A) = \frac{T_{sys}}{\sqrt{\Delta t \, \Delta \nu}}$
- Switching: two measurements, ON-OFF.
  The total uncertainty grows by $\sqrt{2}$

$\Rightarrow \sigma(T_A) = \sqrt{2} \frac{T_{sys}}{\sqrt{\Delta t \, \Delta \nu}}$

- Switching: $\Delta t_{obs} = 2 \, \Delta t$
- $\Rightarrow$ $$\sigma(T_A) = 2 \frac{T_{sys}}{\sqrt{\Delta t_{obs} \, \Delta \nu}}$$

- Example:

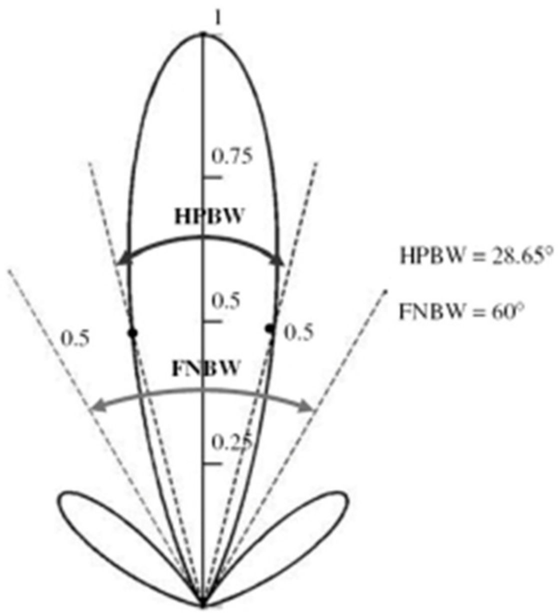
$\Delta t_{obs} = 1$ s , $\Delta \nu = 1$ MHz

$\Rightarrow \sigma(T_A) = 0.001 \, T_{sys}$

$\Rightarrow$ It is possible to observe signal a thousand smaller than the noise (non-astronomical signal).

Observing our astronomical sources

- All done, right?
- We measure voltages, that give T_A,
- ...that gives P
- ...that gives F_ν.
- For a point source, yes.
- But for extended sources
- Beam pattern, resolution, etc.



Antenna beam, beam pattern

- Non-uniform beam pattern $\Rightarrow$ antenna captures power from different points of the sky with different ability.
- The antenna temperature depends on the:
  - intensity of the source (function of position on the sky)   **and also** on the
  - antenna beam or beam pattern (sensitivity of the telescope as a function of position on the sky)
- Beam (or power) pattern: $P_{\mathrm{bm}}(\theta, \phi)$
- Normalized: $P_{\mathrm{bm}}(0, 0)=1$

Antenna beam, beam pattern

- Antenna solid angle, Ω_A, is the integration of the beam power pattern over all angles:

$$\Omega_A = \int P_{bm}(\theta, \phi)\, d\Omega$$ with $d\Omega = \sin\theta \, d\theta \, d\phi$.

- Gives the angular area the telescope responds to.
- Antenna theorem: $\Omega_A = \dfrac{\lambda^2}{A_{eff}}$ (always valid)
- Antenna with circular reflector $\Rightarrow$ beam is axially symmetric $\Rightarrow P_{bm}(\theta, \phi) = P_{bm}(\theta)$
- For uniformly illuminated circular aperture, $P_{bm}(\theta)$ is a Bessel function.

Antenna main beam

- $P_{\text{main beam}}(\theta)$ is a Bessel function, often approximated by a gaussian: $P_{\text{main}}(\theta) = \exp\left[-4\ \ln 2\ \left(\frac{\theta}{\theta_{\text{FWHM}}}\right)^2\right]$
  where, by definition, θ_{FWHM} is the angular resolution.
- For a uniformly illuminated aperture, $\theta_{\text{FWHM}} = 1.02\ \lambda/D$
- For an optimum illuminated aperture, $\theta_{\text{FWHM}} = 1.15\ \lambda/D$

- The *antenna main beam*, or solid angle of the main beam, is : $\Omega_{\text{main}} = \int P_{\text{main beam}}(\theta)\ d\Omega = \frac{1}{\ln 2}\ \frac{\pi}{4}\theta^2{}_{\text{FWHM}}$
- $\Omega_{\text{main}} = \frac{\pi}{4\ \ln 2}\ \theta^2{}_{\text{FWHM}}$

Main beam efficiency

- Example: For a optimum illuminated aperture,

$$\theta_{\rm FWHM} = 1.15\,\lambda/D \quad \Rightarrow \quad \Omega_{\rm main} = \frac{1}{\ln 2}\,\frac{\pi}{4}\,\theta^2_{\rm FWHM} = 1.50\,\frac{\lambda^2}{D^2}$$

- Using $A_{\rm geom} = \frac{\pi}{4} D^2 \;\Rightarrow\; \Omega_{\rm main} = 1.18\,\frac{\lambda^2}{A_{\rm geom}}$
- But $\max(A_{\rm eff}) = 0.82\,A_{\rm geom} \;\Rightarrow\; A_{\rm eff} \le 0.82\,A_{\rm geom}$
- $\Rightarrow \Omega_{\rm main} \le 0.97\,\frac{\lambda^2}{A_{\rm eff}} \quad \Rightarrow \quad \Omega_{\rm main} \le 0.97\,\Omega_{\rm A}$
- The main beam solid angle can be up to 97% of the antenna solid angle, under optimum illumination.
- Define: *main beam efficiency*: $\eta_{\rm mb} = \frac{\Omega_{\rm main}}{\Omega_{\rm A}}$

Aperture efficiency

- Define: *aperture efficiency*: $\eta_A = \dfrac{A_{\mathrm{eff}}}{A_{\mathrm{geom}}}$
- For the optimum illumination case, $\dfrac{\eta_{\mathrm{mb}}}{\eta_A} = 1.18$.

(Note: optical misalignments or out of focus enlarge the main beam $\Rightarrow \theta_{\mathrm{FWHM}} > 1.15\,\lambda/D$)

Power from extended sources

- $P = F_\nu A_{\rm eff}\, \Delta\nu$ but the flux density coming from different directions of the source is not detected with the same efficiency.
- Let the telescope point at $(\theta_0, \phi_0) \Rightarrow P_{\rm bm}(\theta-\theta_0, \phi-\phi_0)$
- Elementary power from (θ, ϕ) is:

$$dP(\theta_0, \phi_0) = I_\nu(\theta, \phi)\, P_{\rm bm}(\theta-\theta_0, \phi-\phi_0)\, d\Omega\ \ A_{\rm eff}\, \Delta\nu$$

$$\Rightarrow \quad P(\theta_0, \phi_0) = \frac{1}{2} \int_{\rm all\ sky} I_\nu(\theta, \phi)\, P_{\rm bm}(\theta-\theta_0, \phi-\phi_0)\, d\Omega\ \ A_{\rm eff}\, \Delta\nu$$

$$\Rightarrow \quad \boxed{T_{\rm A}(\theta_0, \phi_0) = \frac{A_{\rm eff}}{2k} \int_{\rm all\ sky} I_\nu(\theta, \phi)\, P_{\rm bm}(\theta-\theta_0, \phi-\phi_0)\, d\Omega}$$

- $\Rightarrow T_{\rm A}$ is the convolution of the source intensity with the antenna beam: $\boxed{T_{\rm A} = I_\nu \otimes P_{\rm bm}}$

Power from extended sources

- Recall: Brightness temperature, T_B, is a measure of intensity. In the R-J approximation:

$$I_\nu(\theta, \phi) = \frac{2k}{\lambda^2} T_B(\theta, \phi)$$

$$\Rightarrow \boxed{T_A(\theta_0, \phi_0) = \frac{A_{\mathrm{eff}}}{\lambda^2} \int_{\mathrm{all\ sky}} T_B(\theta, \phi)\, P_{\mathrm{bm}}(\theta-\theta_0, \phi-\phi_0)\, d\Omega}$$

- $\Rightarrow$ T_A is the convolution of the source brightness temperature with the antenna beam: $\boxed{T_A = T_B \otimes P_{\mathrm{bm}}}$
- Or equivalently $\boxed{T_A = I_\nu \otimes P_{\mathrm{bm}}}$
- We want I_ν or T_B of the source but we only measure T_A. And T_A is a smeared version of T_B.

Resolved versus unresolved sources

- Extended sources versus point sources.
- More correctly: resolved versus unresolved sources.
- What is a "resolved" source?
  - Angular size > (main) beam size
- What is a "unresolved" source?
  - Angular size < (main) beam size
- What is a "point" source?
  - Angular size << (main) beam size
- A "unresolved" source whose angular size is << beam size is a "point" source.

Point source

- Point source located at (θ_0, ϕ_0) $\Rightarrow$ Only contribution for $\theta=\theta_0, \phi=\phi_0 \quad \Rightarrow \quad P_{bm}(\theta-\theta_0, \phi-\phi_0) \approx P_{bm}(0, 0)=1$

$$\Rightarrow \; T_A(\theta_0, \phi_0) = \frac{A_{eff}}{2k} \int_{source} I_\nu(\theta, \phi) \, d\Omega$$

$$\Rightarrow \; T_A = \frac{A_{eff}}{2k} F_\nu \quad \Rightarrow \quad \boxed{F_\nu = \frac{2k}{A_{eff}} T_A}$$

$\Rightarrow$ For a point source, we get a direct measure of the flux density.

- $\frac{F_\nu}{T_A} = \frac{2k}{A_{eff}}$ is the factor converting antenna temperature to flux density, for a point source.

- The inverse, $\boxed{\frac{T_A}{F_\nu} = \text{DPFU} = \frac{A_{eff}}{2k}}$, (degrees per flux unit), expressed in K Jy^{-1}, is obtained during calibration of the telescope.

Point source

- Unresolved source, but with a known solid angle Ω_{source}.
- $F_\nu = \dfrac{2k}{A_{\text{eff}}} T_A \quad , \quad F_\nu = I_\nu \, \Omega_{\text{source}} \quad , \quad I_\nu = \dfrac{2k}{\lambda^2} T_B$

$$\Rightarrow \quad \frac{2k}{\lambda^2} T_B \Omega_{\text{source}} = \frac{2k}{A_{\text{eff}}} T_A \qquad \Rightarrow \qquad T_A = \frac{A_{\text{eff}}}{\lambda^2} T_B \Omega_{\text{source}}$$

$$\Rightarrow \boxed{T_A = \frac{\Omega_{\text{source}}}{\Omega_A} T_B} \qquad \Rightarrow \quad T_A < T_B$$

- We say that there is *beam dilution* and $\dfrac{\Omega_{\text{source}}}{\Omega_A}$ is the *beam filling factor*.

Unresolved source

- $T_A = \frac{\Omega_{source}}{\Omega_A} T_B$
- If the source just fills the main beam: $\Omega_{source} = \Omega_{main}$

$\Rightarrow \quad T_A = \frac{\Omega_{main}}{\Omega_A} T_B \qquad \Rightarrow \qquad \boxed{T_A = \eta_{mb} T_B}$

Resolved source

- Source is larger than the beam.
- Example 1: source is uniform and fills the main beam but does not extend into the side lobes ($I(\theta, \phi)$ is zero outside the main beam).

$$\Rightarrow T_A(\theta_0, \phi_0) = \frac{A_{\rm eff}}{2k} \int_{\rm all\ sky} I_\nu(\theta, \phi)\, P_{\rm bm}(\theta-\theta_0, \phi-\phi_0)\, d\Omega$$

$\Rightarrow T_A = \frac{A_{\rm eff}}{2k} \Omega_{\rm main} I_\nu \;\Rightarrow$ The antenna temperature is proportional to I_ν not to F_ν.

- If the source is not uniform I_ν → $< I_\nu >$ = average intensity over the main beam.
- $< I_\nu > = \frac{2k}{A_{\rm eff}\Omega_{\rm main}} T_A = \frac{2k}{\lambda^2 \eta_{\rm mb}} T_A$

Resolved source

- $< I_\nu > = \frac{2k}{\lambda^2 \eta_{mb}} T_A$
- Recall that $< I_\nu > \sim F_\nu / \Omega_{main}$
- $< I_\nu >$, the average intensity over the main beam, is commonly referred to as the "*flux density per beam*", in "Jy beam^{-1}", instead of Jy sr^{-1}.
- If a table gives results of observations in Jy/beam, the flux density of each source is:
  - the value quoted, if the source is unresolved
  - the integral (or sum) over the solid angle of the source, if the source is resolved.

Resolved source

- Source is larger than the beam.
- Example 2: source is uniform and fills the entire sky (also, no spillover).

$$\Rightarrow\ T_A(\theta_0, \phi_0) = \frac{A_{eff}}{\lambda^2} \int_{\text{all sky}} T_B(\theta, \phi)\, P_{bm}(\theta-\theta_0, \phi-\phi_0)\, d\Omega$$

$$\Rightarrow\ T_A = \frac{A_{eff}}{\lambda^2} \left[\int_{\text{all sky}} P_{bm}(\theta-\theta_0, \phi-\phi_0)\, d\Omega\right] T_B$$

$$\Rightarrow\ T_A = \frac{A_{eff}}{\lambda^2}\, \Omega_A T_B \quad \Rightarrow\ T_A = T_B$$

- When all sky is filled with uniform radiation of brightness temperature T_B, the measured antenna temperature T_A is equal to this brightness temperature.

Calibration by filling the beam

- Surrounding the telescope (or feed horn) with a source of known T_{B} (a blackbody), allows the measurement of the system temperature.
- This method is used at higher frequencies and also allows the correction of the attenuation of the atmosphere.
- $V = C(T_{\mathrm{A}} + T_{\mathrm{sys}})$

With a "hot load": $V_{\mathrm{HL}} = C(T_{\mathrm{HL}} + T_{\mathrm{sys}})$

With a "cold load": $V_{\mathrm{CL}} = C(T_{\mathrm{CL}} + T_{\mathrm{sys}})$

$$T_{\mathrm{sys}} = \frac{T_{\mathrm{HL}} - T_{\mathrm{CL}}\, y}{y - 1} \qquad \text{with} \qquad y = V_{\mathrm{HL}}/V_{\mathrm{CL}}$$

e.g. T_{CL} = 78 K (liquid N_2); T_{HL} = 293 K (ambient)

Spectral line observations

- Requires a spectrometer at the back-end
- Spectral parameters (fixed for the filter bank):
  - Spectral resolution: width of each channel, $\delta\nu$
  - Total bandwidth: $\Delta\nu = N_{channels}\ \delta\nu$
  - Central IF (frequency of the central channel)
- LO is used to select the RF we want to observe: $\nu_{RF} = \nu_{LO} \pm \nu_{IF}$
- To detect and measure the line: minimum number of 5 channels across the line.
- Line must leave free a sufficient number of channels (not occupy more than ¾ of the bandwidth).

Spectral line observations

- Uncertainty in T_A depends on $\delta\nu$. Substitute: $\Delta\nu \rightarrow \delta\nu$ in the equation of uncertainty:

$$\sigma(T_A) = 2\,\frac{T_{\mathrm{sys}}}{\sqrt{\Delta t_{\mathrm{obs}}\,\delta\nu}}$$

- Spectra as a function of velocity (instead of frequency).
- Conversion based on Doppler formula:

$$\frac{\nu - \nu_0}{\nu_0} = \frac{v}{c}$$ (non-relativistic)

Spectral line observations

- Frequency switching: alternative mode of switching possible because the signal is contained in a narrow range of frequencies.
- Switch in frequency (tuning the LO to a slightly different frequency) instead of moving the telescope or secondary.
- Frequency offset not too large: the OFF spectrum should contain the same noise as the ON spectrum.
- Can double the time on-source, thus requiring less observing time: $\sigma(T_A) = \sqrt{2}\,\frac{T_{sys}}{\sqrt{\Delta t_{obs}\,\delta\nu}}$, not: $2\,\frac{T_{sys}}{\sqrt{\Delta t_{obs}\,\delta\nu}}$

Baseline

- The reference (or comparison, or OFF) signal may not be well-balanced in power. The result is a baseline that is not flat nor located at $T_A=0$.
- Requires "subtracting a baseline".
- Usually in the form of linear function of the frequency.
- Sometimes a sinusoidal baseline appears, due to a standing wave between the feed horn and the primary reflector.

Radio images

- When the angular size of the source is larger than the beam, we may wish to make an image of the source.
- But small number of receivers at the telescope.
- Building image point by point covering the sky area desired.
- Sampling: full-beam versus Nyquist.

Convolution

- Remember: The antenna temperature is the convolution of the brightness of the source with the beam pattern.
- $T_A(\theta_0, \phi_0) = \frac{A_{eff}}{2k} \int_{all\ sky} I_\nu(\theta, \phi)\ P_{bm}(\theta-\theta_0, \phi-\phi_0)\ d\Omega$ or $T_A = I_\nu \otimes P_{bm}$
- The resolution of the image is given by the beam size.
- If the beam pattern is well known, and there is sufficient SNR, some resolution can be recovered by making a deconvolution.
- $\theta_{eff} \sim \frac{\theta_{fundamental}}{\sqrt{SNR}}$

Convolution with a point source

- Point source located at $(\theta_0, \phi_0) \Rightarrow$ Only contribution for $\theta{=}\theta_0, \phi{=}\phi_0 \quad \Rightarrow \quad P_{\mathrm{bm}}(\theta-\theta_0, \phi-\phi_0) = P_{\mathrm{bm}}(0, 0) = 1$

$$T_{\mathrm{A}}(\theta_0, \phi_0) = \frac{A_{\mathrm{eff}}}{2k} \int_{\text{all sky}} I_\nu(\theta, \phi)\, P_{\mathrm{bm}}(\theta-\theta_0, \phi-\phi_0)\, d\Omega \quad \Rightarrow T_{\mathrm{A}} = \frac{A_{\mathrm{eff}}}{2k} F_\nu$$

- Point source located at $(\theta_1, \phi_1) \Rightarrow$ Only contribution for $\theta{=}\theta_1, \phi{=}\phi_1 \quad \Rightarrow \quad P_{\mathrm{bm}}(\theta_1-\theta_0, \phi_1-\phi_0) \neq 1$
- $T_{\mathrm{A}}(\theta_1, \phi_1) = \frac{A_{\mathrm{eff}}}{2k} \int_{\text{all sky}} I_\nu(\theta, \phi)\, P_{\mathrm{bm}}(\theta-\theta_0, \phi-\phi_0)\, d\Omega$

$$\Rightarrow T_{\mathrm{A}}(\theta_1, \phi_1) = \frac{A_{\mathrm{eff}}}{2k} F_\nu\, P_{\mathrm{bm}}(\theta_1-\theta_0, \phi_1-\phi_0)$$

- Let position 1 be arbitrary:

$$T_{\mathrm{A}}(\theta, \phi) = \frac{A_{\mathrm{eff}}}{2k} F_\nu\, P_{\mathrm{bm}}(\theta-\theta_0, \phi-\phi_0)$$

Convolution with a point source

- Let position 1 be arbitrary:

$$T_A(\theta, \phi) = \frac{A_{eff}}{2k} F_\nu P_{bm}(\theta - \theta_0, \phi - \phi_0)$$

- The map looks like the beam pattern. That is, the angular dependence of the map is the same as the antenna beam pattern.
- The peak value in the map gives the flux density of the source.
- This is useful to measure the beam pattern of an antenna.

Calibration of a radio telescope

- Pointing
  - Temperature variations, deformation of the dish, etc.
  - Pointing corrections needed
  - Using a five-point scan map in Az-El.
- Gain, Effective Area and Gain curve (function of El).
  - Different telescope gains (<u>not</u> receiver gain). Needs to be <u>measured</u>.
  - DPFU: Gain $= \frac{A_{\text{eff}}}{2k} = \frac{T_A}{F_\nu}$ (measure this one)
  - Another Gain $= \frac{4\pi}{\Omega_A} = \frac{4\pi A_{\text{eff}}}{\lambda^2}$
  - Aperture efficiency: $\eta_A = \frac{A_{\text{eff}}}{A_{\text{geom}}}$

Calibration of a radio telescope

- Beam pattern
  - By mapping a strong point source
  - Inclusively, side lobes

- Main Beam efficiency
  - Extended source to fill the main beam (but not the error pattern). Use $T_{\mathrm{A}} = \eta_{\mathrm{mb}} T_{\mathrm{B}}$ to measure η_{mb}.
  - For short wavelengths (< 1 cm), use large planets.

Calibration of a radio telescope

- Antenna solid angle
  - $\Omega_A = \frac{\lambda^2}{A_{\mathrm{eff}}}$
- Main beam solid angle
  - $\Omega_{\mathrm{main}} = \frac{1}{\ln 2} \frac{\pi}{4} \theta^2{}_{\mathrm{FWHM}}$
- Main beam efficiency
  - $\eta_{\mathrm{mb}} = \frac{\Omega_{\mathrm{main}}}{\Omega_A}$

IRAM

- *https://www.iram-institute.org*