Vector DM Model

A vector DM model

Dark U(1)$_X$ gauge symmetry: all SM particles are U(1)$_X$ neutral.
New complex scalar field - scalar under the SM gauge group but has unit charge under U(1)$_X$.
Lagrangian invariant under

$$X_\mu \to -X_\mu, \quad \mathbb{S} \to \mathbb{S}^*$$

Forbids kinetic mixing between the SM gauge boson from U(1)$_Y$ and the dark one from U(1)$_X$. The Lagrangian is

$$\mathscr{L} = \mathscr{L}_{SM} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu \mathbb{S})^\dagger (D^\mu \mathbb{S}) - \mu_S^2 \left|\mathbb{S}\right|^2 + \lambda_S \left|\mathbb{S}\right|^4 + \kappa \left|\mathbb{S}\right|^2 H^\dagger H \qquad D_\mu = \partial_\mu + i g_X X_\mu$$

with

$$H = \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}(v_H + h + iG_0) \end{pmatrix} \qquad \mathbb{S} = \frac{1}{\sqrt{2}}(v_S + S + iA)$$

h is the real doublet component, S is the new real scalar component and A is the Goldstone boson related with U(1)$_X$.

P: Find the mass of the new gauge boson.

A vector DM model

With the previous definitions, the masses of the gauge bosons are

$$m_W = \frac{1}{2} g v_H;\ m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v_H;\ m_{DM} = g_X v_S$$

and the masses of the two scalars are

$$m_\pm = \lambda_H v_H^2 + \lambda_S v_S^2 \pm \sqrt{\lambda_H^2 v_H^4 + \lambda_S^2 v_S^4 + \kappa v_H^2 v_S^2 - 2\lambda_H \lambda_S v_H^2 v_S^2}$$

The mass eigenstates fields h_1 and h_2 are obtained from h and S via (and the Goldstone is eaten by the vector DM)

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}$$

I will come back to this model later.

Fermion DM model

A fermion DM model

Let us now build a model with a DM fermion. The Lagrangian is

$$\mathscr{L} = \mathscr{L}_{SM} + V_{SM} - V_{New} + \bar{\chi}(\gamma_\mu \partial^\mu - m_\chi)\chi - i y_\chi P \bar{\chi} \gamma_5 \chi + \text{ scalar kinetic terms}$$

where χ is the new DM fermion for which we impose a Z_2 symmetry $\chi \to -\chi$ that is combined with $P \to P$ and $\phi_2 \to -\phi_2$ leading to the following new potential with two complex scalar doublets and one real singlet.

$$V_{New} = m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.) + \frac{m_S^2}{2} P^2 + \kappa (P \Phi_1^\dagger \Phi_2 + h.c.)$$
$$+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$
$$+ \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + h.c. \right] + \frac{\lambda_6}{4} P^4 + \frac{\lambda_7}{2} (\Phi_1^\dagger \Phi_1) P^2 + \frac{\lambda_8}{2} (\Phi_2^\dagger \Phi_2) P^2$$

We will need and extra Z_2 symmetry $\chi \to -\chi$, to make sure that no other Yukawa terms can be built with the SM fermions.

P: Try to build one of these terms

A fermion DM model

The new dark fermion χ couples to two new fields, that come from the rotation of P and the CP-odd field from the doublet.

$$(a \cos\theta + A \sin\theta)\bar{\chi}\gamma_5\chi$$

In turn, a and A provide the link to the remaining SM particles. So the pseudo scalar acts here as the portal.

P: Could we do this with a scalar instead of a pseudoscalar?

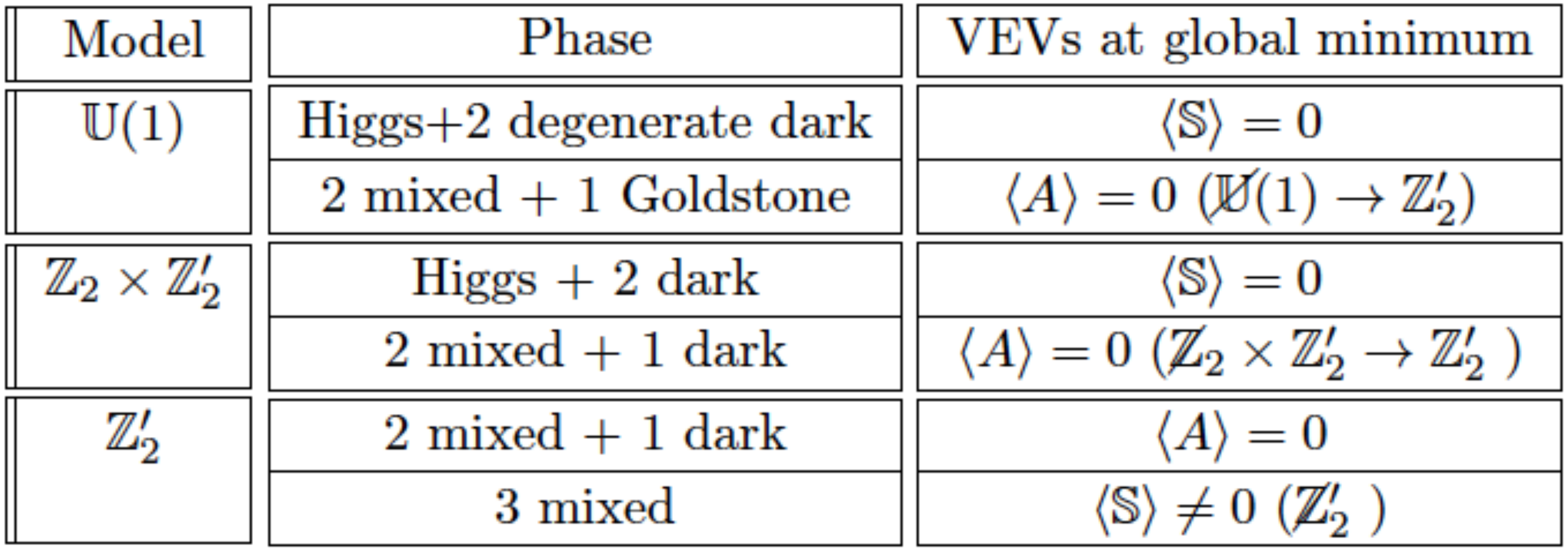
P: If a pseudo scalar is indeed needed, could we do this with one doublet only?

P: What are the diagrams for $pp \to \chi\chi j$? What is the background?

The spin 0 extension - complex

Let us now go back to 5th model on the list

$$\mathscr{L} = \mathscr{L}_{SM} + (D_\mu \mathbb{S})^\dagger (D^\mu \mathbb{S}) + \mu_S^2 \left|\mathbb{S}\right|^2 - \lambda_S \left|\mathbb{S}\right|^4 - \kappa \left|\mathbb{S}\right|^2 H^\dagger H + \mu^2(\mathbb{S}^2 + \mathbb{S}^{*2}) \qquad \mathbb{S} = \frac{1}{\sqrt{2}}(v_S + S + iA)$$



Model	Phase	VEVs at global minimum
$\mathbb{U}(1)$	Higgs+2 degenerate dark	$\langle \mathbb{S} \rangle = 0$
	2 mixed + 1 Goldstone	$\langle A \rangle = 0$ ($\cancel{\mathbb{U}}(1) \to \mathbb{Z}_2'$)
$\mathbb{Z}_2 \times \mathbb{Z}_2'$	Higgs + 2 dark	$\langle \mathbb{S} \rangle = 0$
	2 mixed + 1 dark	$\langle A \rangle = 0$ ($\cancel{\mathbb{Z}_2} \times \mathbb{Z}_2' \to \mathbb{Z}_2'$)
$\mathbb{Z}_2'$	2 mixed + 1 dark	$\langle A \rangle = 0$
	3 mixed	$\langle \mathbb{S} \rangle \neq 0$ ($\cancel{\mathbb{Z}_2'}$)

P: What are the diagrams for $pp \to \chi\chi j$? What is the background?

P: What are the diagrams for $\chi u \to \chi u$? And for $\chi g \to \chi g$?

P: What are the diagrams for $\chi\chi \to hh$? And for $\chi\chi \to \gamma\gamma$?

Rules for extended sectors

Extended scalar sectors

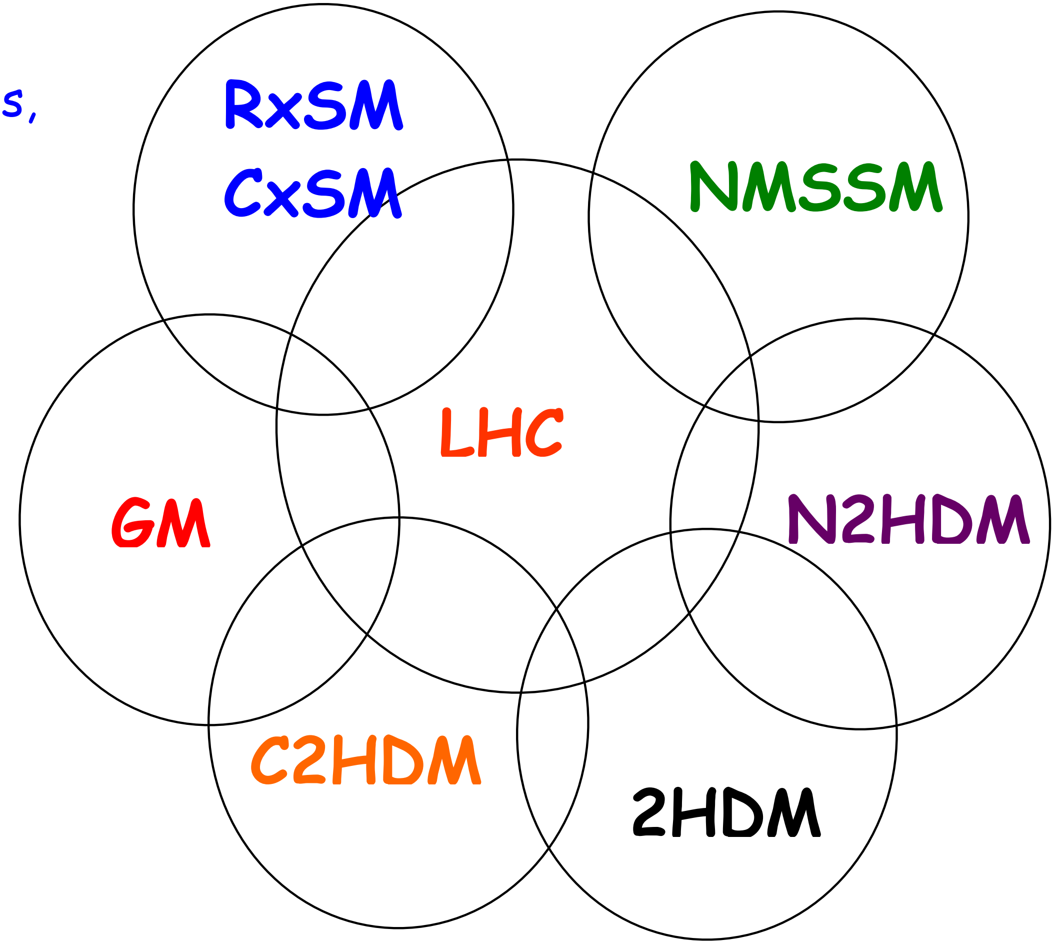
1. **Direct detection of new physics** - Motivate searches at the LHC in simple extensions of the scalar sector – benchmark models for searches.

2. **Indirect detection of new physics** (via measurements of the 125 GeV Higgs couplings)

a) Mixing effects with other Higgs bosons, e.g. singlet, doublet, CP admixtures.

b) How efficiently can the parameter space of these simple extensions be constrained through measurements of Higgs properties? Focus on CP.

c) What are higher order EW corrections (of extended models) good for?



RxSM
CxSM

NMSSM

LHC

GM

N2HDM

C2HDM

2HDM

Many simple model with new and interesting physics

	CxSM (RxSM)	2HDM	C2HDM	N2HDM
Model	SM+Singlet	SM+Doublet	SM+Doublet	2HDM+Singlet
Scalars	$h_{1,2,(3)}$ (CP even)	H, h, A, $H^\pm$	$H_{1,2,3}$ (no CP), $H^\pm$	$h_{1,2,3}$ (CP-even), A, $H^\pm$
Motivation	DM, Baryogenesis	$+\ H^\pm$	+ CP violation	+ ...

Similar neutral Higgs sector but different underlying symmetries

- There is a 125 GeV Higgs (other scalars can be lighter and/or heavier).
- From the 2HDM on, tan $\beta = v_2/v_1$. Also charged Higgs are present.
- Models (except singlet extensions) can be CP-violating.
- They all have $\rho=1$ at tree-level.
- You get a few more scalars (CP-odd or CP-even or with no definite CP)
- In case all neutral scalars mix there will be three mixing angles
- They can have dark matter candidates (or not)

Potential(s)

Potential

$$V = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + h.c.) + \frac{m_S^2}{2}\Phi_S^2$$
$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2) + h.c.\right] + \frac{\lambda_6}{4}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2$$

with fields

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix} \quad \Phi_S = v_S + \rho_S$$

Particle (type) spectrum depends on the symmetries imposed on the model, and whether they are spontaneously broken or not. There are two charged particles and 4 neutral.

The model can be CP violating or not.

magenta $\Longrightarrow$ SM

magenta + blue $\Longrightarrow$ RxSM (also CxSM)

magenta + black $\Longrightarrow$ 2HDM (also C2HDM)

magenta + black + blue + red $\Longrightarrow$ N2HDM

softly broken Z_2 : $\Phi_1 \to \Phi_1;\ \Phi_2 \to -\Phi_2$

softly broken Z_2 : $\Phi_1 \to \Phi_1;\ \Phi_2 \to -\Phi_2;\ \Phi_S \to \Phi_S$

exact Z_2' : $\Phi_1 \to \Phi_1;\ \Phi_2 \to \Phi_2;\ \Phi_S \to -\Phi_S$

- **m^2_{12} and λ_5 real** <u>2HDM</u>
- **m^2_{12} and λ_5 complex** <u>C2HDM</u>

Constraints

- Should contain a SM-like Higgs boson

- Electroweak ρ parameter should be close to 1 (relation between W and Z mass)

$$\rho_{\text{exp}} = 1.0004^{+0.0003}_{-0.0004}$$

$$\rho = \frac{m_W^2}{m_Z^2 \cos\theta_W^2} = \frac{\sum_i \left[4T_i(T_i+1) - Y_i^2\right] |v_i|^2 c_i}{\sum_i 2Y_i^2 |v_i|^2}$$

$$Q = T_3 + Y/2$$

T_i	$SU(2)_L$ **Isospin**
Y_i	**Hypercharge**
v_i	**VEV**
c_i	**1(1/2) for complex (real) representations**

- Perturbative unitarity

- Boundness from below

DM Direct detection

Distribution of Dark Matter in the galaxy

Hard problem - there are only averages over long volumes. There are attempts to measure locally and globally the shape of the Milky Way DM halo.

But what we really need is the kinematic distribution of DM in our solar system.

We assume the Standard Halo Model (SHM) with a density profile of $\rho(r) \sim r^{-2}$. The velocities obey a Boltzmann-Maxwell distribution. The local circular speed of DM is (218-246) Km/s. The velocity distribution is cut at the escape velocity, which is about 530 Km/s.

The prediction for the direct detection of DM on the Earth is separated into a kinematical part involving the velocity distribution and one part that deals with the collision. This allows us to compare different experiments independently of the local DM distribution.

MB distribution - system containing a large number of identical non-interacting, non-relativistic classical particles in thermodynamic equilibrium, the fraction of the particles within an infinitesimal element of the three-dimensional velocity space, cantered on a velocity vector of magnitude v, is

$$f(v)\, d^3v = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}\, d^3v,$$

Direct detection

We assume we have a WIMP that has an electroweak interaction that comes via some portal. Since the DM is coupled to a mediator (in the case of the scalar extension is the Higgs) and the mediator is coupled to the remaining SM particles, there will be an effective DM-SM interaction.

Also, we assume there is a local DM density ρ_0 in which the earth is traveling. The DM stream may interact with a nucleus and transfer a small amount of energy (recoil energy). So far no event was recorded and bounds were set on coupling vs. mass. The differential scattering can be written as

$$\frac{dR(E_R,t)}{dE_R} = N_T \frac{\rho}{m_\chi} \int_{v>v_{min}} v\, f(\vec{v} + \vec{v}_E(t)) \frac{d\sigma(E_R,v)}{dE_R}\, d^3v \qquad [\sigma v n] = m^2 \frac{m}{s} \frac{1}{m^3} = \frac{1}{s}$$

where E_R is the recoil energy, N_T is the number of nuclei, v is the velocity in the rest frame of the experiment, f is the velocity distribution function and v_{min} is the minimum velocity of DM causing a recoil energy. The minimum velocity for elastic scattering is

$$v_{min} = \sqrt{\frac{m_N E_R}{2\mu^2}}, \qquad \mu = \frac{m_N m_\chi}{m_N + m_\chi}$$

where m_N is the nucleon mass.

Direct detection

The differential rate can further be divided in a spin-dependent (SD) and a spin-independent (SI) part. The time integrated differential cross section is then written as

$$\frac{\sigma(E_R, v)}{dE_R} = \frac{m_N}{2\mu^2 v^2}\left(\sigma^{SI} F^2_{SI}(E_R) + \sigma^{SD} F^2_{SD}(E_R)\right)$$

where F are nuclear form factors. The DM velocity is non-relativistic, $v/c \approx 10^{-3}$, and therefore the recoil energies are low (order KeV) and the momentum transfer is of order GeV. This in turn means that nuclei cannot be treated as point-like in the scattering process with DM. The cross section with a target nucleus is

$$\sigma_i^{SI} = \frac{\mu_i}{\pi} \left| Z_i g_p^{SI} + (A_i - Z_i) g_n^{SI} \right|^2 \left| F_i(q) \right|^2$$

where i indicates the material and Z and A are the proton and mass numbers, respectively.

Now we need to find a way to link the quarks to the nucleons.

Let us see how exactly we can do this.

Intermission - EFTs

Let us go back to the Fermi theory of weak interactions, with Lagrangian

$$\mathscr{L}_{int} = \frac{G_F}{\sqrt{2}} \sum_{i,j} \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i \;\; \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i$$

In the electroweak theory this interaction would have been written as

$$\mathscr{L}_{int} = \frac{g^2}{8} \sum_{i,j} \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i \;\; \frac{-1}{q^2 - m_W^2} \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i$$

And in the limit $q^2 \ll m_W^2$ we can write

$$\mathscr{L}_{int} \approx \frac{g^2}{8m_W^2} \sum_{i,j} \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i \;\; \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i \quad (q^2 \ll m_W^2)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

We say that we have matched the Wilson coefficient $G_F/\sqrt{2}$ to the coefficient of the actual model. This yields G_F = 1.17 × 10^{-5} GeV^{-2} . Theory works well for and energy well below the W boson mass. At higher energies one should use the proper electroweak theory.

Direct detection at LO

- Write the effective Lagrangian

Wilson coefficients

$$\mathcal{L}^{\rm eff} = \sum_{q=u,d,s} \mathcal{L}_q^{\rm eff} + \mathcal{L}_G^{\rm eff}$$

$$\mathcal{L}_q^{\rm eff} = f_q \chi_\mu \chi^\mu m_q \bar{q} q + \frac{g_q}{m_\chi^2} \chi^\rho i\partial^\mu i\partial^\nu \chi_\rho \mathcal{O}^q_{\mu\nu}\,,$$

$$\mathcal{L}_G^{\rm eff} = f_G \chi_\rho \chi^\rho G^a_{\mu\nu} G^{a\,\mu\nu}\,, \quad \mathcal{O}^q_{\mu\nu} = \frac{1}{2} \bar{q} i \left(\partial_\mu \gamma_\nu + \partial_\nu \gamma_\mu - \frac{1}{2} \not{\partial} \right) q\,.$$

- Define the nucleon matrix elements

f_{T_q} denotes the fraction of the nucleon mass that is due to light quark q (lattice)

$$\langle N | m_q \bar{q} q | N \rangle = m_N f^N_{T_q}$$

$$-\frac{9\alpha_S}{8\pi} \langle N | G^a_{\mu\nu} G^{a,\mu\nu} | N \rangle = \left(1 - \sum_{q=u,d,s} f^N_{T_q} \right) m_N = m_N f^N_{TG}$$

$$\langle N(p) | \mathcal{O}^q_{\mu\nu} | N(p) \rangle = \frac{1}{m_N} \left(p_\mu p_\nu - \frac{1}{4} m_N^2 g_{\mu\nu} \right) \left(q^N(2) + \bar{q}^N(2) \right)\,,$$

SHIFMAN, VAINSHTEIN, ZAKHAROV, PLB78 443 (1978)

fraction of the nucleon momentum carried by the quarks (PDFs)

- And calculate the cross section

$$\sigma_N = \frac{1}{\pi} \left(\frac{m_N}{m_\chi + m_N} \right)^2 \left| f_N \right|^2 .$$

$$f_N / m_N = \sum_{q=u,d,s} f_q f^N_{T_q} + \sum_{q=u,d,s,c,b} \frac{3}{4} \left(q^N(2) + \bar{q}^N(2) \right) g_q - \frac{8\pi}{9\alpha_S} f^N_{TG} f_G\,.$$

And now we need to get all the Wilson coefficients f_q, g_q, f_G at the order we are working at

Direct detection at LO for scalars

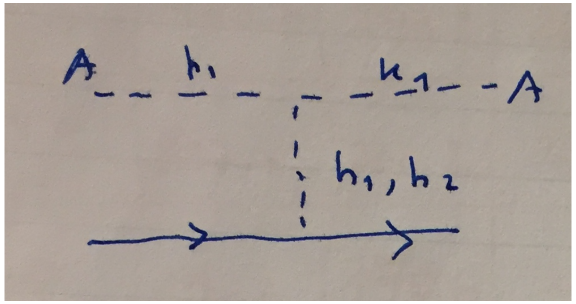
- Write the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \sum_q C_S^q \mathcal{O}_S^q + C_S^g \mathcal{O}_S^g + \sum_q C_T^q \mathcal{O}_T^q$$

$$\mathcal{O}_S^q = m_q \chi^2 \bar{q} q \,,$$
$$\mathcal{O}_S^g = \frac{\alpha_s}{\pi} \chi^2 G^a_{\mu\nu} G^{a\mu\nu} \,,$$
$$\mathcal{O}_T^q = \frac{1}{m_\chi^2} \chi i\partial^\mu i\partial^\nu \chi \mathcal{O}^q_{\mu\nu} \,.$$

- Quark contributions

Exchanged momentum very small



$$\mathcal{A}_{\rm gen} = \sum_i C_{\chi\chi h_i} C_{qqh_i} \frac{1}{q^2 - m_{h_i}^2} \bar{u}(\mathbf{p}) u(\mathbf{p}+\mathbf{q}) \xrightarrow{q^2 \to 0} -\sum_i C_{\chi\chi h_i} C_{qqh_i} \frac{1}{m_{h_i}^2} \bar{u}(\mathbf{p}) u(\mathbf{p})$$

Assuming scalar-like couplings we can write

$$\mathcal{L}_{\rm eff} \supset -\sum_i \frac{C_{\chi\chi h_i} C_{qqh_i}}{2 m_{h_i}^2} \chi\chi \bar{q} q$$

Term in the effective Lagrangian

And so the Wilson coefficient is

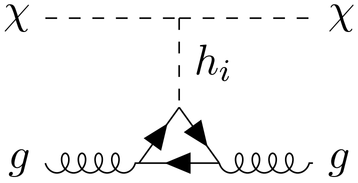
$$C_S^q \supset -\sum_i \frac{C_{\chi\chi h_i} C_{qqh_i}}{2 m_q m_{h_i}^2}$$

There can be additional contributions to the quark operators generated through other diagrams, even though at tree level the t-channel exchange is the only topology contributing to this operator in the models under investigation.

Direct detection at LO for scalars

- Gluon contributions

$$m_Q \bar{Q} Q \to -\frac{\alpha_s}{12\pi} G^a_{\mu\nu} G^{a\mu\nu}$$



- This transformation can be used to write

$$\frac{f_N^{\rm LO}}{m_N} = f_q^{\rm LO} \left[\sum_{q=u,d,s} f^N_{T_q} + \sum_{q=c,b,t} \frac{2}{27} f^N_{T_G} \right]$$

- And so the final cross section is

$$\sigma_N = \frac{1}{\pi} \left(\frac{m_N}{m_\chi + m_N} \right)^2 |f_N|^2 .$$

And for normalisation the Wilson coefficient in a model with two scalars is

$$f_q = \frac{1}{2} \frac{g g_\chi}{m_W} \frac{\sin(2\alpha)}{2} \frac{m^2_{h_1} - m^2_{h_2}}{m^2_{h_1} m^2_{h_2}} m_\chi \,, \quad q = u, d, s, c, b, t$$

Nuclear form factors

We here present the numerical values for the nuclear form factors defined in Eq. (4.59). The values of the form factors for light quarks are taken from `micrOmegas` [75]

$$f^p_{T_u} = 0.01513\,, \quad f^p_{T_d} = 0.0.0191\,, \quad f^p_{T_s} = 0.0447\,, \tag{A.99a}$$

$$f^n_{T_u} = 0.0110\,, \quad f^n_{T_d} = 0.0273\,, \quad f^n_{T_s} = 0.0447\,, \tag{A.99b}$$

which can be related to the gluon form factors as

$$f^p_{T_G} = 1 - \sum_{q=u,d,s} f^p_{T_q}\,, \qquad f^n_{T_G} = 1 - \sum_{q=u,d,s} f^n_{T_q}\,. \tag{A.100}$$

The needed second momenta in Eq. (4.59) are defined at the scale $\mu = m_Z$ by using the `CTEQ` parton distribution functions [76],

$$u^p(2) = 0.22\,, \qquad \bar{u}^p(2) = 0.034\,, \tag{A.101a}$$

$$d^p(2) = 0.11\,, \qquad \bar{d}^p(2) = 0.036\,, \tag{A.101b}$$

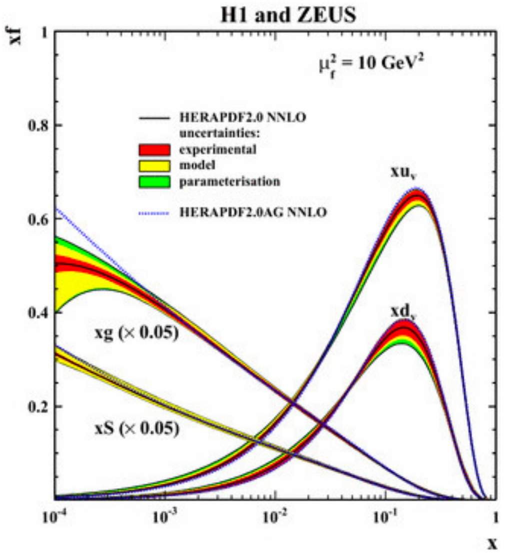
$$s^p(2) = 0.026\,, \qquad \bar{s}^p(2) = 0.026\,, \tag{A.101c}$$

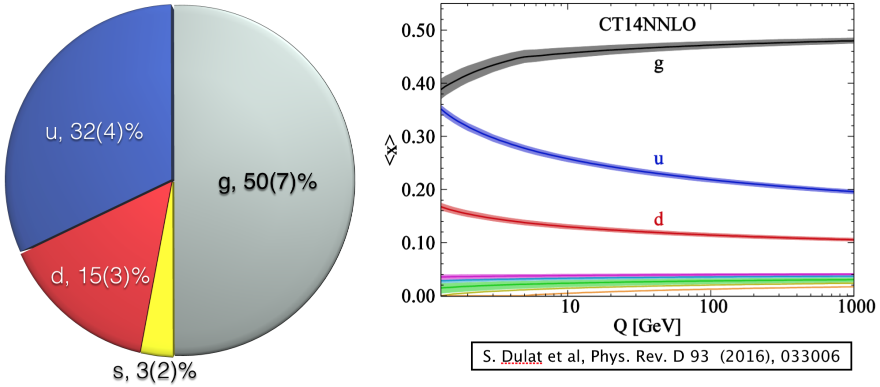
$$c^p(2) = 0.019\,, \qquad \bar{c}^p(2) = 0.019\,, \tag{A.101d}$$

$$b^p(2) = 0.012\,, \qquad \bar{b}^p(2) = 0.012\,, \tag{A.101e}$$

where the respective second momenta for the neutron can be obtained by interchanging up- and down-quark values.

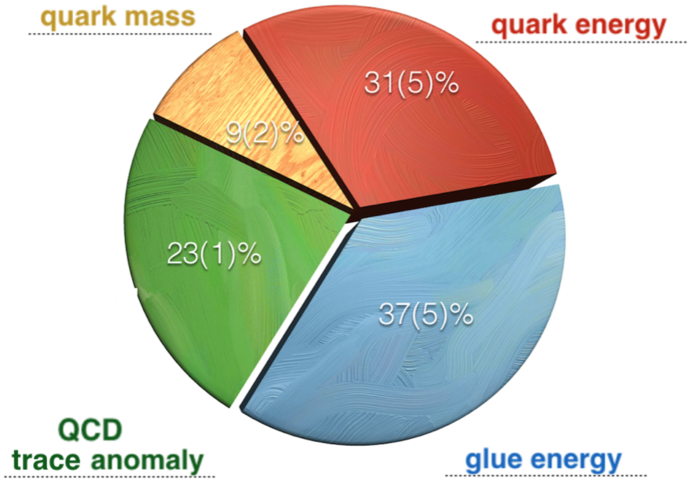
Nuclear form factors





S. Dulat et al, Phys. Rev. D 93 (2016), 033006

Figure 1. The contributions of different quark flavors and glue to the proton momentum fraction. The left panel shows the lattice results renormalized in the $\overline{\text{MS}}$ scheme at 2 GeV with 1-loop perturbative calculation and proper normalization of the glue. The experimental values are illustrated in the right panel, as a function of the $\overline{\text{MS}}$ scale. Our results agree with the experimental values at 2 GeV.



Yang et al., arXiv:1710.09011v1 (2018)

Figure 2. The pie chart of the proton mass decomposition, in terms of the quark mass, quark energy, glue field energy and trace anomaly.

The spin 0 extension - real

The SM is extended by an extra real scalar singlet S. The most general Lagrangian we can write is

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - aS - bS^2 - cS^3 - dS^4 - \kappa_1 S H^\dagger H - \kappa_2 S H^\dagger H - \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

And with a Z_2 symmetry $S \to -S$, the potential reduces to

$$V_N = bS^2 + dS^4 + \kappa_1 S^2 H^\dagger H + \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Let us consider the solution (for the minimum)

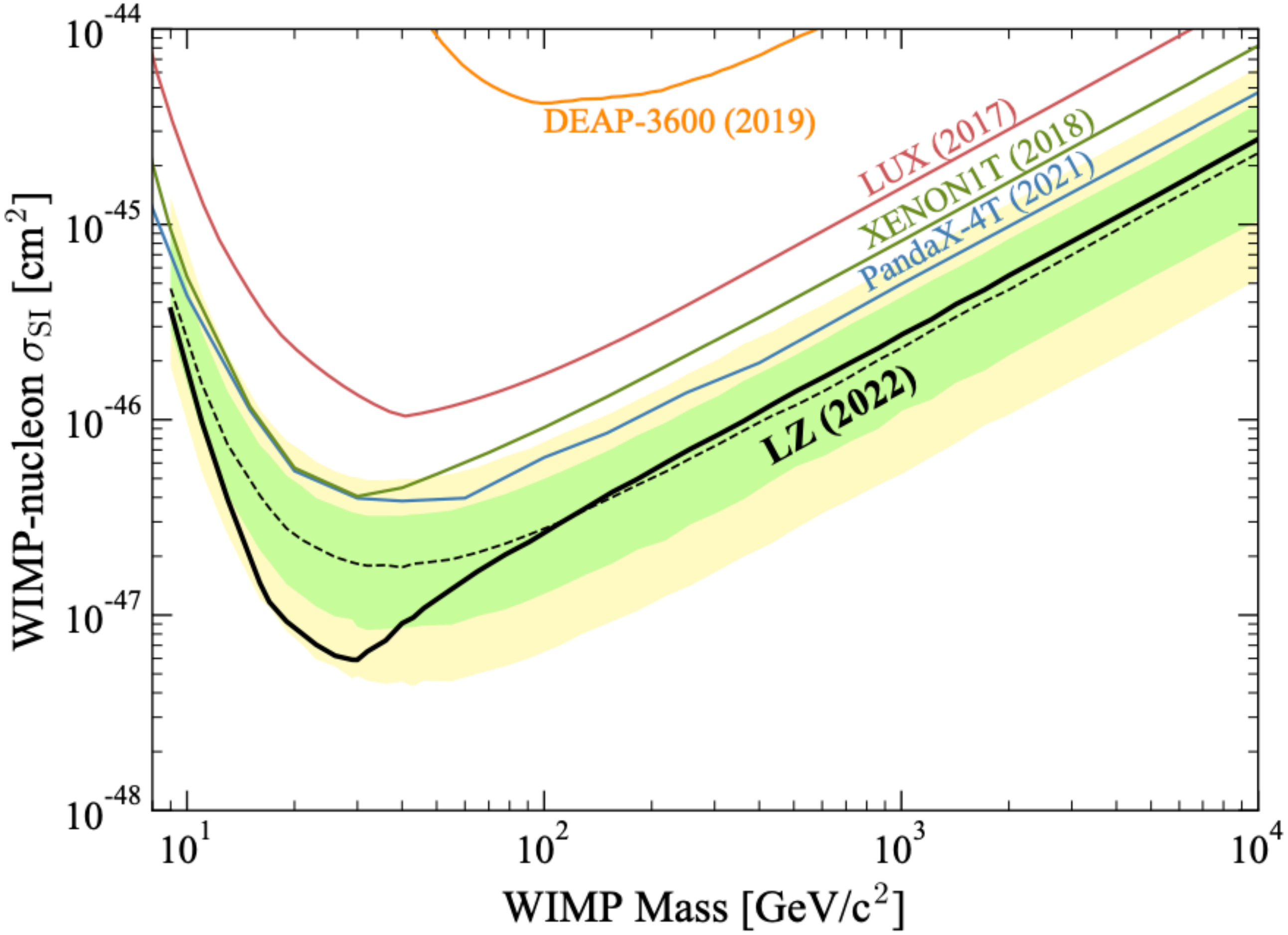
$$S = 0;\ h^2 = -\mu^2/(2\lambda);$$

P: Collect the relevant couplings for direct detection.

P: Calculate the amplitude.

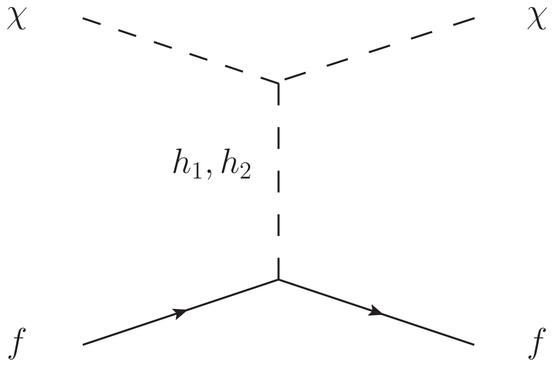
DD measurements

This is what we have to compare to.



Back to the complex spin zero extension

Let us now consider the same process but in the complex extension. The relevant pieces of the Lagrangian are



$$\mathcal{L} \supset \frac{v_s}{2}\chi^2\left(\kappa_{\chi\chi h_1}\, h_1 + \kappa_{\chi\chi h_2}\, h_2\right)$$

$$\begin{aligned}\kappa_{\chi\chi h_1} &= +\, m_{h_1}^2/v_s^2\ \sin\theta \\ \kappa_{\chi\chi h_2} &= -\, m_{h_2}^2/v_s^2\ \cos\theta\end{aligned}$$

And

$$\mathcal{L} \supset -(h_1\cos\theta + h_2\sin\theta)\sum_f \frac{m_f}{v}\bar{f}f$$

P: What is now the amplitude in the limit of zero exchanged momentum?

At higher orders - just to have a look

Direct detection at NLO

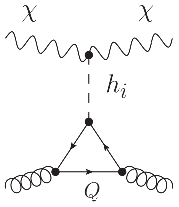
- The NLO EW SI cross section can be obtained using the one-loop form factor

$$\frac{f_N^{\rm NLO}}{m_N} = \sum_{q=u,d,s} f_q^{\rm NLO} f_{T_q}^N + \sum_{q=u,d,s,c,b} \frac{3}{4}\left(q(2)+\bar q(2)\right) g_q^{\rm NLO} - \frac{8\pi}{9\alpha_S} f_{T_G}^N f_G^{\rm NLO}$$

with the Wilson coefficients at one-loop given by

$$\begin{aligned} f_q^{\rm NLO} &= f_q^{\rm vertex} + f_q^{\rm med} + f_q^{\rm box} \\ g_q^{\rm NLO} &= g_q^{box} \\ f_G^{\rm NLO} &= -\frac{\alpha_S}{12\pi} \sum_{q=c,b,t} \left(f_q^{\rm vertex} + f_q^{\rm med}\right) + f_G^{\rm top} \end{aligned}$$

Box diagrams contribute to the two different quark operators.

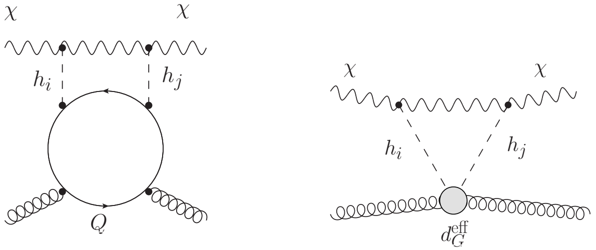


The LO form factor is given by

$$\frac{f_N^{\rm LO}}{m_N} = f_q^{\rm LO}\left[\sum_{q=u,d,s} f_{T_q}^N + \sum_{q=c,b,t} \frac{2}{27} f_{T_G}^N\right]$$

And the cross section at one-loop is

$$\sigma_N = \frac{1}{\pi}\left(\frac{m_N}{m_\chi + m_N}\right)^2 \left[|f_N^{\rm LO}|^2 + 2{\rm Re}\left(f_N^{\rm LO} f_N^{\rm NLO*}\right)\right]$$



Ertas, Kahlhoefer, JHEP06 052 (2019)

Abe, Fujiwara, Hisano, JHEP 02, 028 (2019)

$$\mathcal{L}^{hhGG} = \frac{1}{2} d_G^{\rm eff} h_i h_j \frac{\alpha_S}{12\pi} G^a_{\mu\nu} G^{a\,\mu\nu}$$

$$f_G^{\rm top} = \left(d_G^{\rm eff}\right)_{ij} C_\triangle^{ij} \frac{-\alpha_S}{12\pi}\,.$$

$$f_q = \frac{1}{2}\frac{g g_\chi}{m_W}\frac{\sin(2\alpha)}{2}\frac{m_{h_1}^2 - m_{h_2}^2}{m_{h_1}^2 m_{h_2}^2} m_\chi\,, \quad q = u,d,s,c,b,t$$

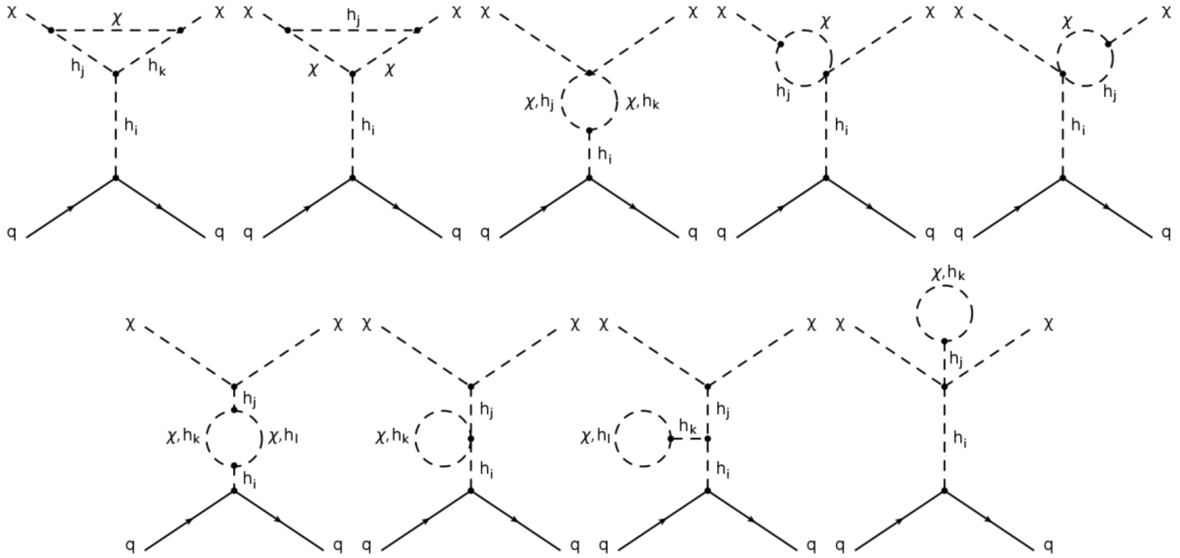
Direct detection at NLO

S2HDM - Now the SM is extended by one doublet and a complex singlet. There is an extra doublet compared to the previous model.

$$\mathscr{V} = \sum_{ij} m_{ij}^2 \phi_i^\dagger \phi_j + \sum_{ijkl} \lambda_{ijkl}\, \phi_i^\dagger \phi_j \phi_k^\dagger \phi_l + \sum_{ij} \kappa_{ij} \left|\mathbb{S}\right|^2 \phi_i^\dagger \phi_j - \mu_S^2 \left|\mathbb{S}\right|^2 + \lambda_S \left|\mathbb{S}\right|^4 \; + \mu^2(\mathbb{S}^2 + \mathbb{S}^{*2})$$

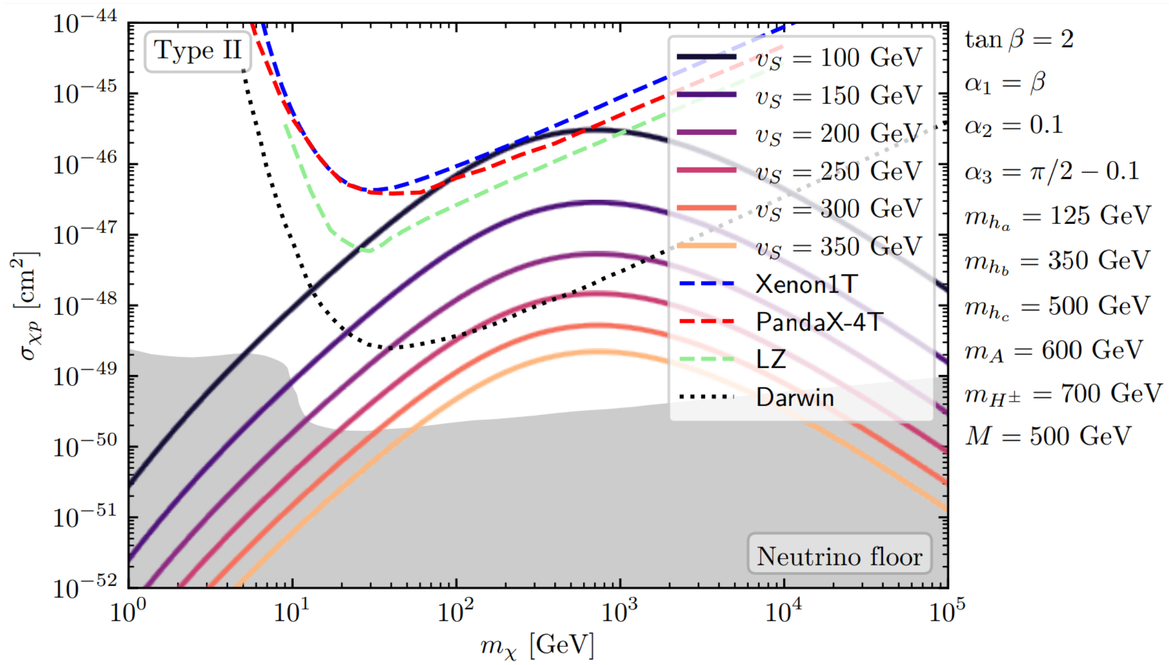
Extra particles: 2 CP-even scalars, 2 charged scalars and 1 CP-odd scalar and a DM particle. Free parameters $m_{h_{1,2,3}}, m_A, m_\chi, \alpha_{1,2,3}, \tan\beta, m_{12}^2, v_S$.

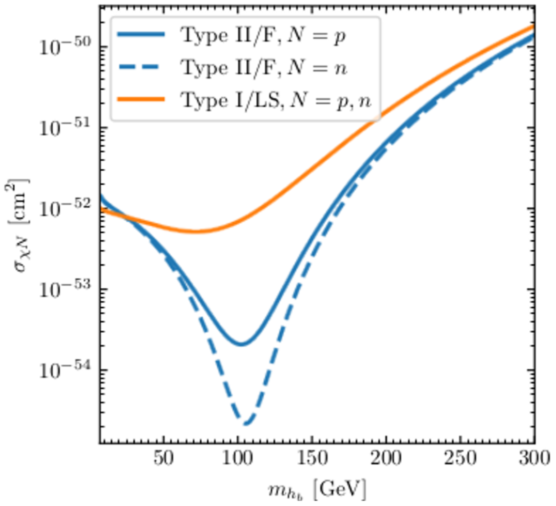
These models can lead to tree-level flavour changing neutral currents. These are very constrained by experiment. To solve this problem one usually forces the Yukawa Lagrangian to be invariant under a Z_2 symmetry. This leads to 4 possible Yukawa Lagrangians (the way scalars are combined with fermions).



Diagrams that survive. Same type of diagrams as for the CxSM but with more particles in the loop.

Direct detection at NLO





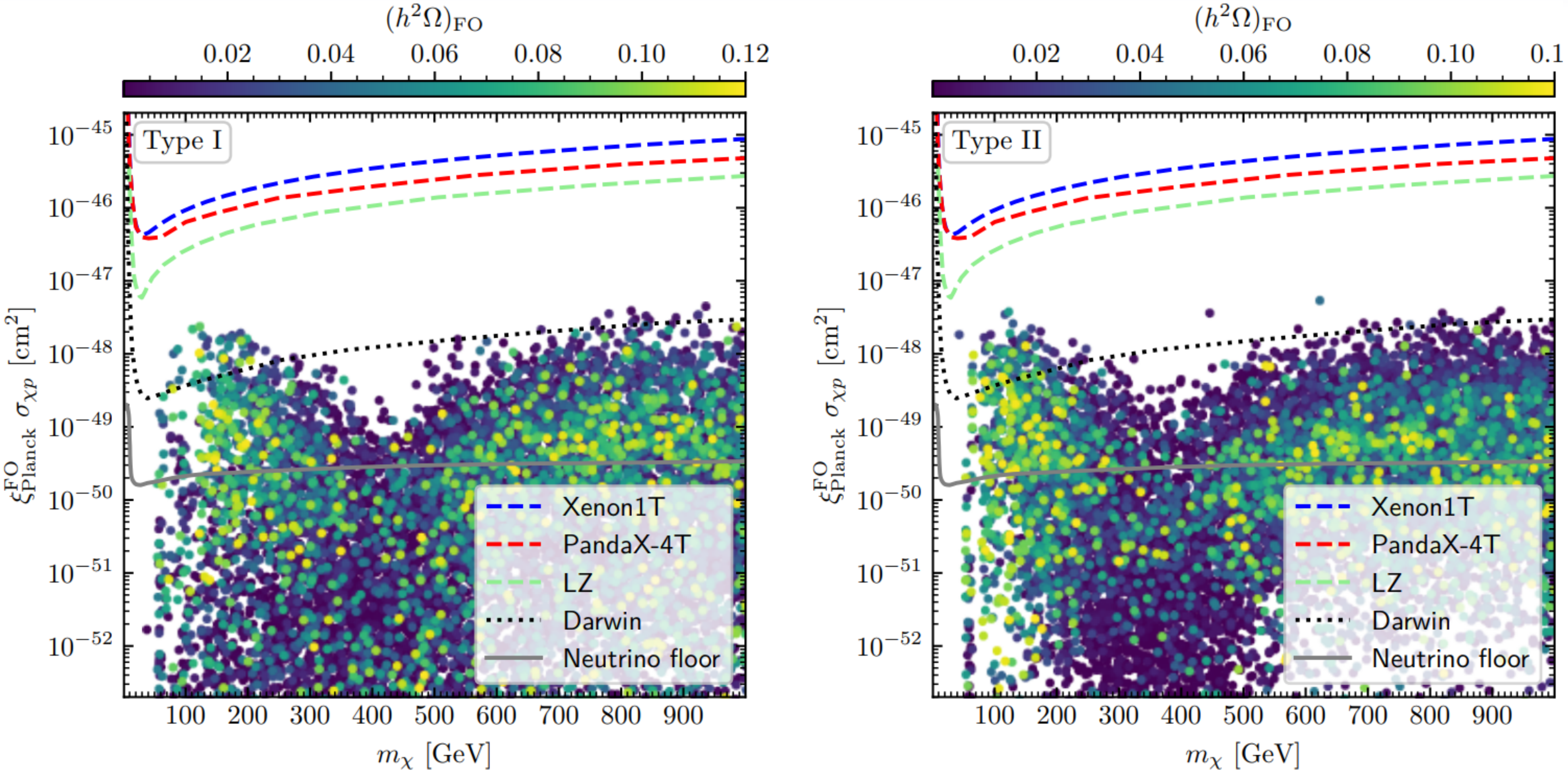
Type dependent blind-spots

Here we just fixed all input parameters except for the VEV of the singlet. The behaviour is similar for all values of the singlet VEV but as the VEV gets smaller a larger mass region in the WIMP region is excluded.

We also show Darwin as an example of some future projection. This is the total cross section.

Direct detection at NLO

Experimental prospect for direct detection in Types I and II



Type	m_{h_a}	$m_{h_b}, m_{h_c}, m_A, m_\chi$	$m_{H^\pm}$	$\alpha_{1,2,3}$	$\tan\beta$	M	v_S
I	125.09	[30,1000]	[150,1000]	$[-\pi/2, \pi 2]$	[1.5,10]	[20, 1000]	[30,1000]

Type	m_{h_a}	m_{h_b,m_A}	$m_{H^\pm}$	$m_{h_c,\chi}$	$\alpha_{1,2,3}$	$\tan\beta$	M	v_S
II	125.09	[200,1000]	[650,1000]	[30,1000]	$[-\pi/2, \pi 2]$	[1.5,10]	[450, 1000]	[30,1000]

Scalar DM but more interesting

Peculiar Scalar extensions of the SM

Some models have negligible dark matter direct detection (DD) cross section at zero momentum transfer (at leading order). Barely affected by direct detection bounds.

True for models with a pNG dark matter candidate with origin in a potential of the form

$$\mathscr{V} = \sum_{ij} m_{ij}^2 \phi_i^\dagger \phi_j + \sum_{ijkl} \lambda_{ijkl}\, \phi_i^\dagger \phi_j \phi_k^\dagger \phi_l + \sum_{ij} \kappa_{ij} \left|\mathbb{S}\right|^2 \phi_i^\dagger \phi_j - \mu_S^2 \left|\mathbb{S}\right|^2 + \lambda_S \left|\mathbb{S}\right|^4 + \mu^2(\mathbb{S}^2 + \mathbb{S}^{*2})$$

with

$$\phi_i = \begin{pmatrix} c^\pm \\ \frac{1}{\sqrt{2}}(v_i + a_i + i b_i) \end{pmatrix} \qquad \mathbb{S} = \frac{1}{\sqrt{2}}(v_S + S + iA)$$

which is a model with N Higgs Doublet Model plus a complex singlet.

The potential is invariant under

$$\mathbb{S} \to \mathbb{S}^* \qquad \text{Stabilises } A$$

and without the red term it is also invariant under

$$\mathbb{S} \to e^{i\alpha} \mathbb{S}$$

The soft breaking term gives mass to the pNG dark matter.

One doublet and one complex singlet (CxSM)

The SM is extended by an extra complex scalar singlet $\mathbb{S}$ which has a global U(1) symmetry

$$\mathbb{S} \to e^{i\alpha}\mathbb{S}$$

Softly break dark U(1) symmetry to the residual Z_2 symmetry in one of the singlet components

$$\mathscr{L} = \mathscr{L}_{SM} + (D_\mu \mathbb{S})^\dagger (D^\mu \mathbb{S}) + \mu_S^2 \left|\mathbb{S}\right|^2 - \lambda_S \left|\mathbb{S}\right|^4 - \kappa \left|\mathbb{S}\right|^2 H^\dagger H + \mu^2(\mathbb{S}^2 + \mathbb{S}^{*2}) \qquad \mathbb{S} \to \mathbb{S}^*$$

SM + dark matter candidate A + a new scalar that mixes with the CP-even field in the doublet such that

$$m_\pm = \lambda_H v_H^2 + \lambda_S v_S^2 \pm \sqrt{\lambda_H^2 v_H^4 + \lambda_S^2 v_S^4 + \kappa v_H^2 v_S^2 - 2\lambda_H \lambda_S v_H^2 v_S^2}$$

The mass eigenstates fields h_1 and h_2 are obtained from h and S via

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}$$

The conditions for the potential to be bounded from below are the same for the two models

$$\lambda_H > 0, \qquad \lambda_S > 0, \qquad \kappa > -2\sqrt{\lambda_H \lambda_S}.$$

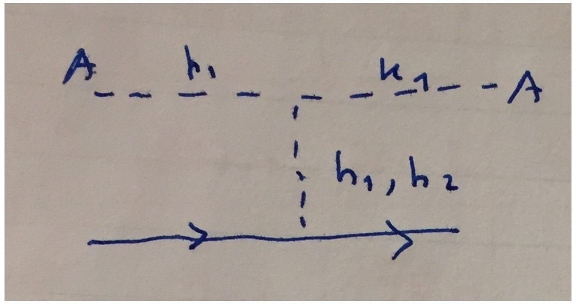
The scalar mass matrix is

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_S & 0 \\ \kappa v v_S & 2\lambda_S v_S^2 & 0 \\ 0 & 0 & -4\mu^2 \end{pmatrix}$$

$$\boxed{m_{DM} = -4\mu^2}$$

One doublet and one complex singlet (CxSM)

The amplitude for the DM direct detection cross section



$$i\mathscr{M} \sim \sin\alpha \cos\alpha \left(\frac{i m_{h_2}^2}{t - m_{h_2}^2} - \frac{i m_{h_1}^2}{t - m_{h_1}^2} \right) \left(\frac{-i m_f}{v} \right) \bar{u}_f(k_2) u_f(p_2) \sim 0 \qquad (t \to 0)$$

And it vanishes for zero momentum transfer. Why? Going back to the Lagrangian,

$$\mathscr{L} = \mathscr{L}_{SM} + (D_\mu \mathbb{S})^\dagger (D^\mu \mathbb{S}) + \mu_S^2 \left| \mathbb{S} \right|^2 - \lambda_S \left| \mathbb{S} \right|^4 - \kappa \left| \mathbb{S} \right|^2 H^\dagger H + \mu^2 (\mathbb{S}^2 + \mathbb{S}^{*2}) \qquad \mathbb{S} \to \mathbb{S}^*$$

Writing

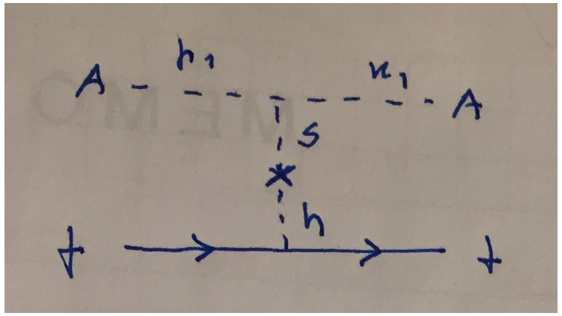
$$\mathbb{S} = \frac{v_S + S}{\sqrt{2}} e^{i \frac{A}{v_S}} \quad \Rightarrow \quad V_{soft} = -\mu^2 (v_S + S)^2 \cos\left(\frac{2A}{v_S} \right) = -\mu^2 (v_S + S)^2 \left(1 - \frac{2A^2}{v_S^2} \right) + \ldots$$

Including the kinetic term leads to the following Lagrangian interaction

$$\mathscr{L}_{SA^2} = = \frac{1}{2 v_S} (\partial^2 S) A^2 - \frac{1}{v_S} S A (\partial^2 + m_A^2) A$$

First term proportional to p² of S and the second term vanishes when the DM particle is on-shell. Amplitude is proportional to p² with A on-shell.

One doublet and one complex singlet (CxSM)



$$i\mathscr{M} \sim \left(\frac{-it}{v_S}\right)\frac{i}{t-m_S^2}(-i2\lambda_{SH}vv_S)\frac{i}{t-m_h^2}\left(\frac{-im_f}{v}\right)\bar{u}_f(k_2)u_f(p_2)$$

Which vanishes when t = 0

Note however if other soft breaking terms are added

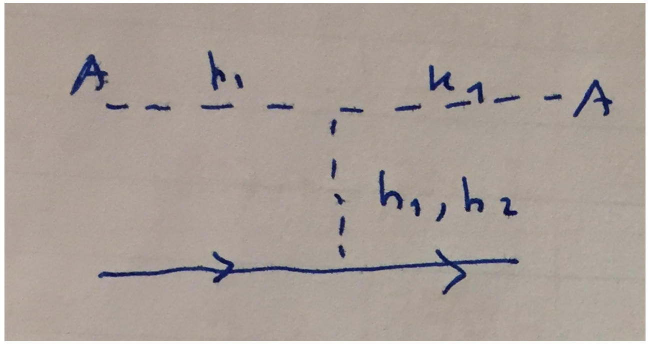
$$V'_{soft} = -\kappa_1^3(\mathbb{S}+\mathbb{S}^*) - \kappa_2|\mathbb{S}|^2(\mathbb{S}+\mathbb{S}^*) - \kappa_3(\mathbb{S}^3+\mathbb{S}^{*3})$$

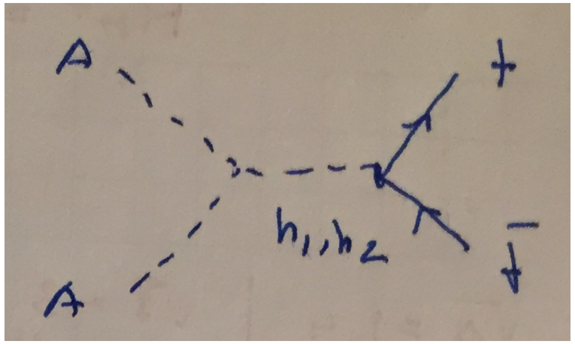
the cancellation is lost except for fine-tuned values of the couplings

$$\kappa_1^3 = \frac{1}{2}(\kappa_2 + 9\kappa_3)v_S^2$$

One doublet and one complex singlet (CxSM)

Note that the cancellation does not happen in scattering





<u>INDEPENDENT PARAMETERS</u>

$$m_{DM};\ \sin\alpha;\ m_2;\ v_S \longrightarrow$$ Singlet VEV

Mass of the DM particle

Mixing angle between doublet and singlet (real)

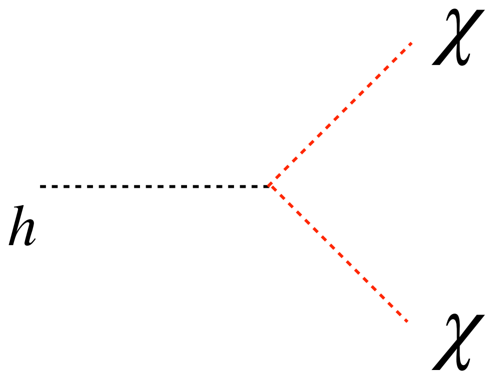
Mass of the second scalar

There is obviously a 125 GeV Higgs (other scalar can be lighter or heavier).

Experimental and theoretical constraints included.

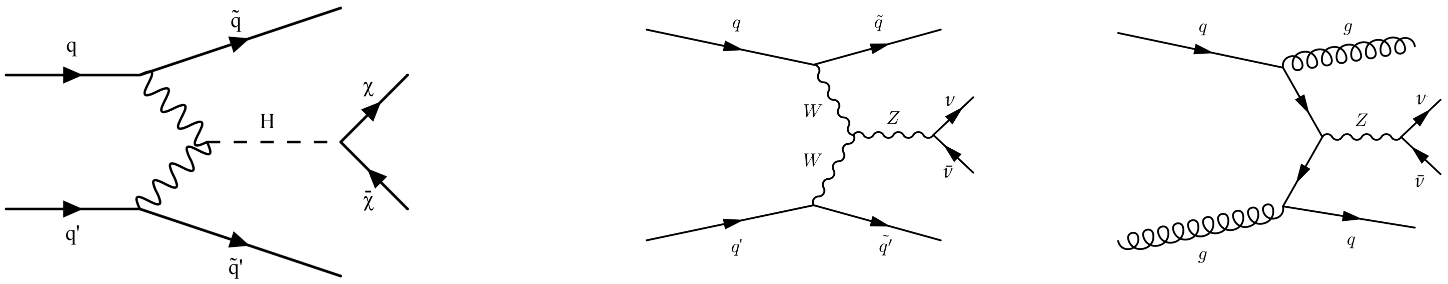
Back to colliders
- the Higgs invisible width

Back to colliders - Higgs invisible width



If the dark matter particle has a mass that is below half of Higgs mass, the Higgs can decay to a dark matter pair.

One of the many on-going searches is

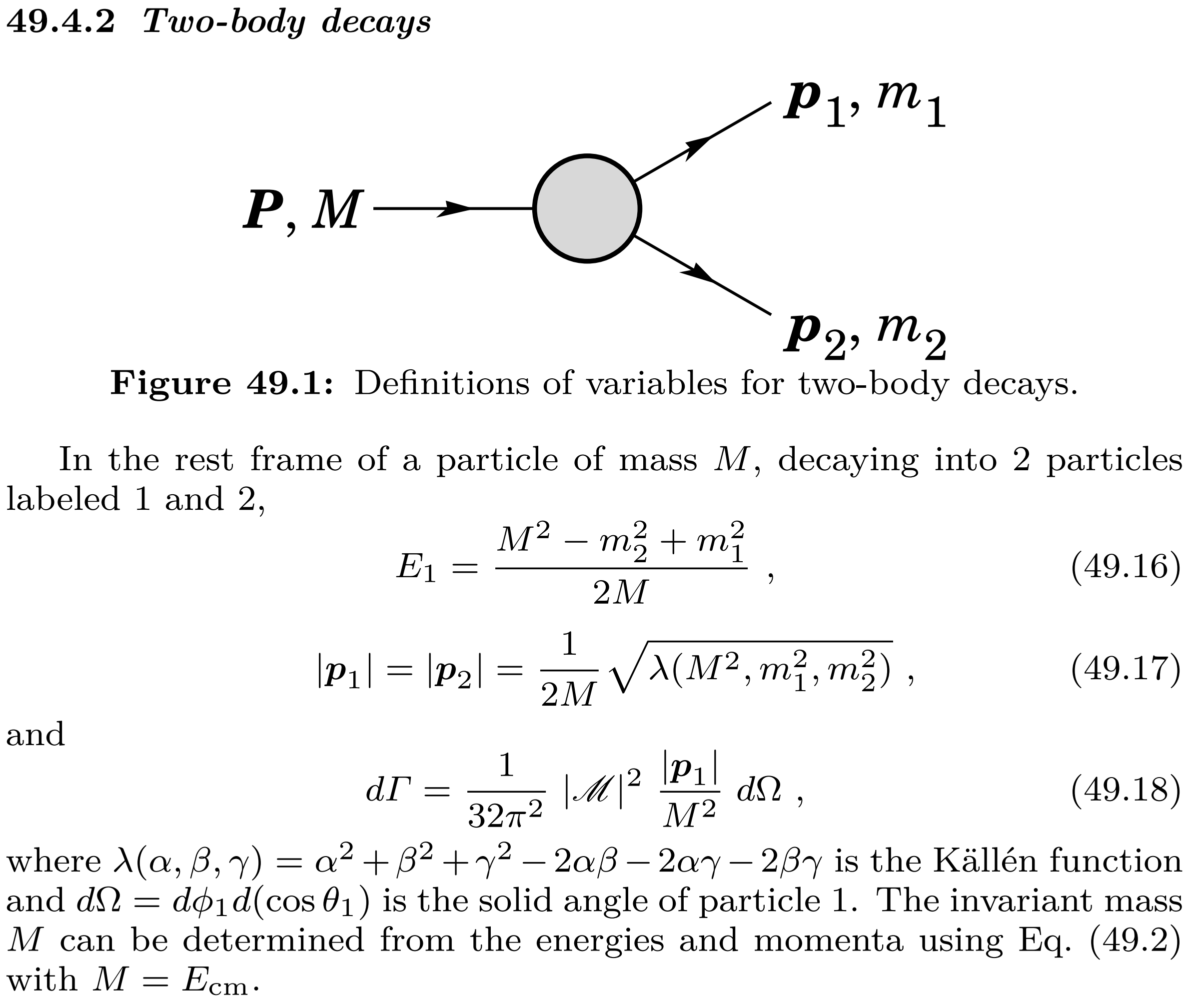


The result gives us a bound on the BR of the Higgs to invisible

$$BR(h \to \chi\chi) = \frac{\Gamma(h \to \chi\chi)}{\Gamma_T(h)} \qquad \Gamma_T(h) \approx 4.6\ MeV$$

Back to colliders - Higgs invisible width

The width is calculated using



49.4.2 *Two-body decays*

Figure 49.1: Definitions of variables for two-body decays.

In the rest frame of a particle of mass M, decaying into 2 particles labeled 1 and 2,

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \, , \tag{49.16}$$

$$|\boldsymbol{p}_1| = |\boldsymbol{p}_2| = \frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)} \, , \tag{49.17}$$

and

$$d\Gamma = \frac{1}{32\pi^2} \, |\mathscr{M}|^2 \, \frac{|\boldsymbol{p}_1|}{M^2} \, d\Omega \, , \tag{49.18}$$

where $\lambda(\alpha, \beta, \gamma) = \alpha^2 + \beta^2 + \gamma^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma$ is the Källén function and $d\Omega = d\phi_1 d(\cos\theta_1)$ is the solid angle of particle 1. The invariant mass M can be determined from the energies and momenta using Eq. (49.2) with $M = E_{\rm cm}$.

Back to colliders - Higgs invisible width

Now calculate the invisible BR for the three models

Scalar - The SM is extended by an extra real scalar singlet S, with a Z_2 symmetry $S \to -S$

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - V_N + V_{SM} \qquad V_N = bS^2 + dS^4 + \kappa_1 S^2 H^\dagger H + \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Let us consider the solution (for the minimum)

$$S = 0;\ h^2 = -\mu^2/(2\lambda);$$

Vector - Dark U(1)$_X$ gauge symmetry: all SM particles are U(1)$_X$ neutral.

$$\mathscr{L} = \mathscr{L}_{SM} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + (D_\mu \mathbb{S})^\dagger (D^\mu \mathbb{S}) + \mu_S^2 \left|\mathbb{S}\right|^2 - \lambda_S \left|\mathbb{S}\right|^4 - \kappa \left|\mathbb{S}\right|^2 H^\dagger H \qquad D_\mu = \partial_\mu + i g_X X_\mu$$

with

$$H = \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}(v_H + h + iG_0) \end{pmatrix} \qquad \mathbb{S} = \frac{1}{\sqrt{2}}(v_S + S + iA)$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}$$

Back to colliders - Higgs invisible width

Now choose a DM mass of 40 GeV and calculate the bound on the portal coupling

$$BR(h \to \chi\chi) = \frac{\Gamma(h \to \chi\chi)}{\Gamma_T(h)} \qquad \Gamma_T(h) \approx 4.6\ MeV$$

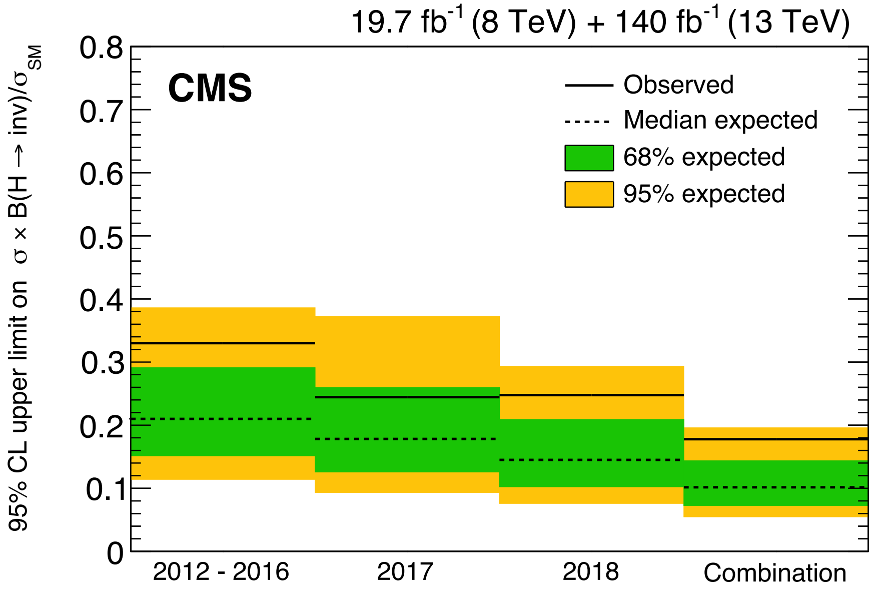


Figure 11: Observed and expected 95% CL upper limits on $(\sigma_H/\sigma_H^{SM})\mathcal{B}(H \to \text{inv})$ for all data-taking years considered, as well as their combination, assuming an SM Higgs boson with a mass of 125.38 GeV.