Fermionic Dark Matter

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Motivation and minimality

- Dark matter (DM) requires physics beyond the SM.
- Aim: minimal, renormalizable, gauge-invariant extension with fermionic DM.
- A singlet fermion $\chi \sim (1, 1, 0)$ has no renormalizable couplings to SM fields.
- Introduce a singlet scalar mediator $S \sim (1, 1, 0)$ to connect χ to the Higgs sector.

Field content and symmetries

- New fields: a gauge-singlet fermion χ (DM) and a real gauge-singlet scalar S (mediator).
- No $\mathbb{Z}_2$ symmetry imposed on S (so odd terms in S are allowed).
- To ensure DM stability one may still impose $\chi \to -\chi$ (or a conserved dark fermion number); here we simply assume χ is stable.

Most general renormalizable Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - V(H,S) - y\, S\, \bar{\chi}\chi.$$

The most general renormalizable, gauge-invariant scalar potential (no $\mathbb{Z}_2$ on S) is

$$V(H,S) = -\mu_H^2\, H^\dagger H + \lambda_H (H^\dagger H)^2 + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{3}\mu_3 S^3 + \frac{1}{4}\lambda_S S^4 + \mu_1 S + \mu_{HS}\, S\, H^\dagger H + \frac{1}{2}\lambda_{HS}\, S^2\, H^\dagger H. \quad (1)$$

All parameters are taken real.

EWSB and field expansions

Take

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \qquad S = v_s + s,$$

with $v \simeq 246\ \mathrm{GeV}$ and singlet vev v_s.
Then

$$H^\dagger H = \frac{(v+h)^2}{2}.$$

Tree-level minimization conditions

Insert the vevs into the potential and impose

$$\left.\frac{\partial V}{\partial h}\right|_0 = 0, \qquad \left.\frac{\partial V}{\partial s}\right|_0 = 0,$$

equivalently $\partial V/\partial v = 0$ and $\partial V/\partial v_s = 0$.
Using $H^\dagger H = v^2/2$ and $S = v_s$ at the minimum, one finds:

$$0 = -\mu_H^2 + \lambda_H v^2 + \mu_{HS} v_s + \frac{1}{2}\lambda_{HS} v_s^2, \tag{2}$$

$$0 = \mu_1 + \mu_S^2 v_s + \mu_3 v_s^2 + \lambda_S v_s^3 + \frac{1}{2}\mu_{HS} v^2 + \frac{1}{2}\lambda_{HS} v^2 v_s. \tag{3}$$

It is often convenient to trade μ_H^2 and μ_1 for (v, v_s) via (2)–(3).

CP-even mass matrix in detail

Define fluctuations (h, s) about the minimum. The CP-even mass-squared matrix is

$$\mathcal{M}^2_{ij} = \left.\frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j}\right|_{h=s=0}, \qquad (\varphi_1, \varphi_2) = (h, s).$$

A direct expansion gives:

$$\mathcal{M}^2_{hh} = -\mu_H^2 + 3\lambda_H v^2 + \mu_{HS} v_s + \frac{1}{2}\lambda_{HS} v_s^2, \tag{4}$$

$$\mathcal{M}^2_{hs} = v\left(\mu_{HS} + \lambda_{HS} v_s\right), \tag{5}$$

$$\mathcal{M}^2_{ss} = \mu_S^2 + 2\mu_3 v_s + 3\lambda_S v_s^2 + \frac{1}{2}\lambda_{HS} v^2. \tag{6}$$

Using the minimization condition (2), the (hh) entry simplifies to

$$\mathcal{M}^2_{hh} = 2\lambda_H v^2.$$

So

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & v(\mu_{HS} + \lambda_{HS} v_s) \\ v(\mu_{HS} + \lambda_{HS} v_s) & \mu_S^2 + 2\mu_3 v_s + 3\lambda_S v_s^2 + \frac{1}{2}\lambda_{HS} v^2 \end{pmatrix}.$$

Diagonalization, mixing angle, eigenvalues

Define mass eigenstates (h_1, h_2) by

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix},$$

with h_1 usually taken as the 125 GeV SM-like Higgs.

Mixing angle:

$$\tan 2\theta = \frac{2\mathcal{M}^2_{hs}}{\mathcal{M}^2_{hh} - \mathcal{M}^2_{ss}}.$$

Eigenvalues:

$$m^2_{1,2} = \frac{1}{2}\left[\mathcal{M}^2_{hh} + \mathcal{M}^2_{ss} \mp \sqrt{\left(\mathcal{M}^2_{hh} - \mathcal{M}^2_{ss}\right)^2 + 4(\mathcal{M}^2_{hs})^2}\right].$$

Couplings and DM interactions

- Yukawa term: $-yS\bar{\chi}\chi = -y(v_s + s)\bar{\chi}\chi$.
- Physical DM mass (tree-level) becomes

$$m_\chi^{\rm phys} = m_\chi + y v_s.$$

- In terms of mass eigenstates:

$$\mathcal{L} \supset -y\left(\sin\theta\, h_1 + \cos\theta\, h_2\right)\bar{\chi}\chi.$$

- Couplings of h_1 to SM fields are rescaled by $\cos\theta$ relative to the SM.

The 2HDM+a

Extend the SM with:

- two Higgs doublets (Φ_1, Φ_2),
- a pseudoscalar singlet a_0.

Key feature: mixing between CP-odd states of the 2HDM and the singlet a_0.

Field content

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\eta_i) \end{pmatrix}, \qquad i = 1, 2, \qquad v^2 = v_1^2 + v_2^2, \quad \tan\beta = \frac{v_2}{v_1}.$$

Singlet pseudoscalar: a_0.

Lagrangian structure and kinetic terms

$$\mathcal{L} = \mathcal{L}_{\rm kin} - V(\Phi_1, \Phi_2, a_0) + \mathcal{L}_{\rm Yukawa}.$$

$$\mathcal{L}_{\rm kin} = \sum_{i=1}^{2} (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + \frac{1}{2}(\partial_\mu a_0)(\partial^\mu a_0),$$

$$D_\mu = \partial_\mu + igW_\mu^a T^a + ig' Y B_\mu.$$

2HDM potential + singlet portal

$$
\begin{aligned}
V_{\mathrm{2HDM}} &= m_{11}^2\,\Phi_1^\dagger\Phi_1 + m_{22}^2\,\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) \\
&\quad + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \qquad (7) \\
&\quad + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\
&\quad + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right], \qquad (8)
\end{aligned}
$$

$$
V_a = \frac{1}{2}m_a^2 a_0^2 + \frac{\lambda_a}{4}a_0^4, \qquad (9)
$$

$$
V_{\mathrm{portal}} = i\kappa\, a_0\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right), \qquad (10)
$$

$$
V = V_{\mathrm{2HDM}} + V_a + V_{\mathrm{portal}}. \qquad (11)
$$

CP-odd mixing and DM sector

Define physical CP-odd states (A, a) from $(A_{\rm 2HDM}, a_0)$:

$$\begin{pmatrix} A \\ a \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A_{\rm 2HDM} \\ a_0 \end{pmatrix}.$$

Dirac fermion DM:

$$\mathcal{L}_{\rm DM} = \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi - i y_\chi a_0 \bar{\chi}\gamma_5\chi \quad \Rightarrow \quad \mathcal{L} \supset -i y_\chi (\cos\theta\, a + \sin\theta\, A)\bar{\chi}\gamma_5\chi.$$