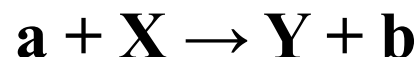


NUCLEAR REACTIONS

In a typical nuclear reaction, a heavy nucleus **X** is bombarded by light particles **a**, giving rise to other heavy nucleus/nuclei **Y** and other light particle(s) **b**:



In the energy ranges of interest in medicine, the number of protons and neutrons is *conserved*. Nuclear reactions can be classified in many different ways, in particular depending on the energy needed to trigger them, on their *energy balance* and on the *cross-section* of the reaction (the probability of the reaction occurring).

- Any radioactive decay is an example of a nuclear reaction.



ENERGY BALANCE OF NUCLEAR REACTIONS

The energy released in radioactive decays or in reactions involving several nuclei comes from the conversion of part of the mass into energy, in accordance with Einstein's relativity. In general, the energy **Q** (sometimes called the *Q-value*) exchanged in a radioactive decay or in a nuclear reaction is given by

$$Q = [(total\ initial\ mass) - (total\ final\ mass)] c^2$$

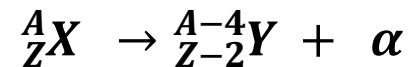
The simplest way to do the calculation is to convert all the masses involved into *MeV*.

A value of $Q > 0$ means *released energy*, whereas $Q < 0$ means that the reaction *absorbed energy*.

- In radioactive decays, we **always have** $Q > 0$ – a decay always releases energy. It cannot consume energy because it is a spontaneous process.
- Nuclear reactions with $Q > 0$ release energy – these are called **exothermic** or **exoenergetic**.
- Nuclear reactions with $Q < 0$ consume energy – these are called **endothermic** or **endoenergetic**.

The energy Q must be calculated using the masses **OF THE NUCLEI** involved in the reaction/decay. However, with the correct addition of electron masses, the calculation is done with *atomic masses*, which are easier to obtain.

- **ALPHA DECAY:** in a generic alpha decay, we have



where α , recall, is a helium nucleus. The *Q-value* of this reaction is therefore

$$Q_\alpha = [m_N({}^A_ZX) - m_N({}^{A-4}_{Z-2}Y) - m_N({}^4_2He)] c^2$$

To go from nuclear masses to atomic masses, we add and subtract Z electron masses, with which we “construct” the atomic masses:

$$\begin{aligned} Q_\alpha &= [m_N({}^A_ZX) + Z m_e - m_N({}^{A-4}_{Z-2}Y) - m_N({}^4_2He) - Z m_e] c^2 \\ &= [\{m_N({}^A_ZX) + Z m_e\} - \{m_N({}^{A-4}_{Z-2}Y) + (Z - 2) m_e\} - \{m_N({}^4_2He) + 2 m_e\}] c^2 \end{aligned}$$

Note that the terms in square brackets are exactly the atomic masses of X, Y and the alpha particle. Hence, in terms of atomic masses

$$Q_\alpha = [m({}^A_ZX) - m({}^{A-4}_{Z-2}Y) - m({}^4_2He)] c^2$$

- **BETA-MINUS DECAY:** in a β^- generic decay, we have



The antineutrino ν has a mass close to zero, which can be neglected. The *Q-value* of this reaction is therefore

$$Q_{\beta^-} = [m_N({}^A_ZX) - m_N({}^A_{Z+1}Y) - m_e] c^2$$

To go from nuclear masses to atomic masses, once again we add and subtract Z electron masses, with which we “construct” the atomic masses:

$$\begin{aligned} Q_{\beta^-} &= [m_N({}^A_ZX) + Z m_e - m_N({}^A_{Z+1}Y) - m_e - Z m_e] c^2 \\ &= [\{m_N({}^A_ZX) + Z m_e\} - \{m_N({}^A_{Z+1}Y) + (Z + 1) m_e\}] c^2 \end{aligned}$$

Note that the mass of the electron emitted in the decay is “absorbed” into the $Z + 1$ electron masses needed to make up the atomic mass of Y. Once again, the terms in square brackets are exactly the atomic masses of X and Y. Hence, in terms of atomic masses, the energy released in a β^- decay is simply

$$Q_{\beta^-} = [m({}^A_ZX) - m({}^A_{Z+1}Y)] c^2$$

- **BETA-PLUS DECAY:** in a β^+ generic decay, we have



Once again, the neutrino ν has a mass close to zero, which can be neglected. The Q -value of this reaction is therefore

$$Q_{\beta^+} = [m_N({}^A_Z X) - m_N({}^A_{Z-1} Y) - m_e] c^2$$

Note that the positron (e^+) has exactly the same mass as an electron. To go from nuclear masses to atomic masses, once again we add and subtract Z electron masses, with which we “construct” the atomic masses:

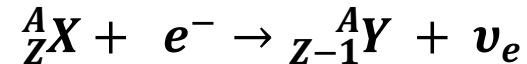
$$\begin{aligned} Q_{\beta^+} &= [m_N({}^A_Z X) + Z m_e - m_N({}^A_{Z-1} Y) - m_e - Z m_e] c^2 \\ &= [\{m_N({}^A_Z X) + Z m_e\} - \{m_N({}^A_{Z-1} Y) + (Z - 1) m_e\} - 2 m_e] c^2 \end{aligned}$$

Since the final nucleus Y has atomic number $Z - 1$, two electron masses are “left over” that would be needed to make up the atomic mass of Y . The terms in square brackets are exactly the atomic masses of X and Y . Hence, in terms of atomic masses, the energy released in a β^+ decay is given by

$$Q_{\beta^+} = [m({}^A_Z X) - m({}^A_{Z-1} Y) - 2 m_e] c^2$$

We see that β^+ decay can only occur *when the difference between the atomic masses of the initial and final nuclei is greater than two electron masses*. Only then will it be possible to have energy release in the decay.

- **ELECTRON CAPTURE:** in a generic electron capture (ϵ) decay, we have



The calculation of the energy released in this process depends very strongly on the initial energy of the electron (which is in an electron shell close to the nucleus, and is captured by it), and therefore varies for each decay considered. Letting E_{e^-} be the *binding energy* (**NEGATIVE**) of the innermost-shell electron of X , we have that:

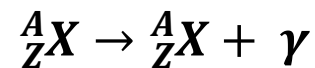
$$\begin{aligned} Q_\epsilon &= [E_{e^-} + m_e + m_N({}^A_ZX) + Z m_e - m_N({}^A_{Z-1}Y) - Z m_e] c^2 \\ &= [E_{e^-} + \{m_N({}^A_ZX) + Z m_e\} - \{m_N({}^A_{Z-1}Y) + (Z - 1) m_e\}] c^2 \end{aligned}$$

And therefore, in terms of atomic masses, we obtain

$$Q_\epsilon = [E_{e^-} + m({}^A_ZX) - m({}^A_{Z-1}Y)] c^2$$

It can thus be seen that electron capture decay (which always occurs in parallel with β^+ decays, *when these exist*) is “easier” to occur, and is not limited by the two-electron-mass difference threshold.

- **GAMMA DECAY:** in a generic γ decay, we have



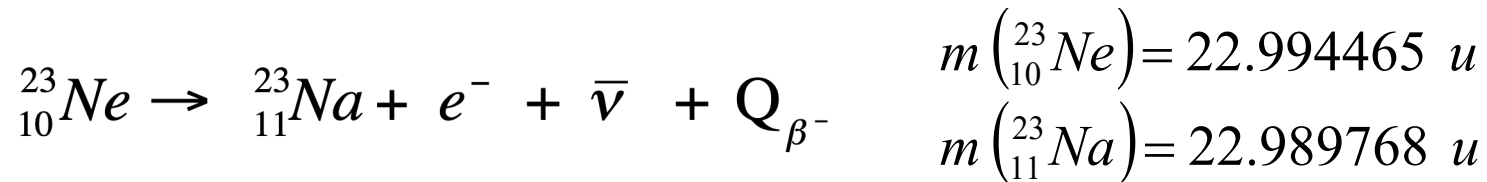
The energy of the emitted photon depends very strongly on the structural details of each nucleus involved, and cannot be calculated in a simple way – there is therefore no general formula.

It is possible, in advanced nuclear models (which describe the energy levels allowed for the nucleons of a given nucleus by solving equations analogous to the Schroedinger equation), to obtain the energies of the photons emitted in gamma decays.

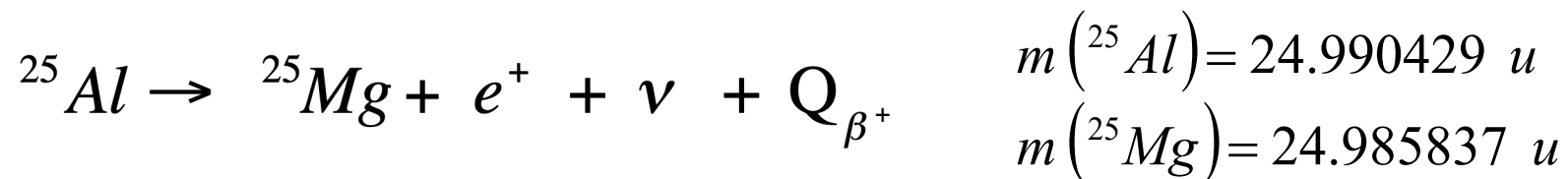
Examples



$$Q = (229.032073 \text{ u} - 225.023205 \text{ u} - 4.002603 \text{ u}) \times 931.502 \text{ MeV/u} = 5.84 \text{ MeV}$$



$$Q_{\beta^{-}} = (22.994465 - 22.989768)(931.502) = 4.38 \text{ MeV}$$

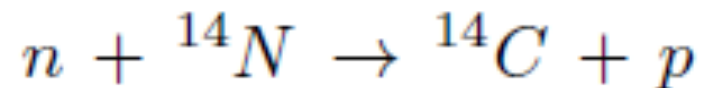


$$Q_{\beta^{+}} = (24.990429 - 24.985837 - 2 \times 5.485803 \times 10^{-4})(931.502) = 3.26 \text{ MeV}$$

NUCLEAR REACTIONS

- The number of protons and neutrons **is conserved** in the reaction, and must be equal in the **reaction products** and in the **reactants**.
- To calculate the *Q-value*, the number of electron masses needed to make up the corresponding atomic masses must be added and subtracted.
- Since **neutrons** are neutral, they do not form atoms when isolated – if isolated neutrons appear in a reaction, we must consider their masses alone – these neutrons will not be part of any atom.
- Isolated protons, together with their associated electrons, will form atomic masses of **hydrogen**. Similarly, **alpha** particles plus **two** electron masses will form the atomic mass of **helium**.

EXAMPLES: in the reaction forming carbon-14 in the atmosphere, nitrogen-14 is bombarded by neutrons produced by cosmic rays, such that



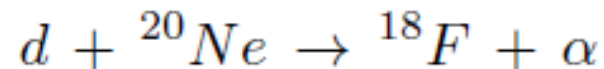
Note the conservation of the number of protons and neutrons.

To calculate the energy involved in this reaction, we start as usual with the nuclear masses, and “construct” the atomic masses:

$$\begin{aligned}
 Q &= [m_n + m_N(^{14}\text{N}) - m_N(^{14}\text{C}) - m_p] c^2 \\
 &= [m_n + m_N(^{14}\text{N}) + 7m_e - m_N(^{14}\text{C}) - m_p - 7m_e] c^2 \\
 &= [m_n + \{m_N(^{14}\text{N}) + 7m_e\} - \{m_N(^{14}\text{C}) + 6m_e\} - \{m_p + m_e\}] c^2 \\
 &= [m_n + m(^{14}\text{N}) - m(^{14}\text{C}) - m(\text{H})] c^2
 \end{aligned}$$

The last line is the expression to use to calculate the value of Q – it is expressed in atomic masses, which are tabulated. Note that the neutron remains isolated, but the proton, together with an electron, gives rise to an atomic mass of hydrogen, $m(\text{H})$.

Another example: one of the ways to produce fluorine-18 is from neon atoms via the reaction



whose energy is given by

$$\begin{aligned}
 Q &= [m_N(d) + m_N({}^{20}\text{Ne}) - m_N({}^{18}\text{F}) - m_N({}^4\text{He})] c^2 \\
 &= \dots \\
 &= [m({}^2\text{H}) + m({}^{20}\text{Ne}) - m({}^{18}\text{F}) - m({}^4\text{He})] c^2
 \end{aligned}$$