

Why is an absorbing boundary needed to calculate the survival probability?

Consider a Brownian particle starting at $x = 0$, with a target at $x = a > 0$. The **survival probability** $S_a(t)$ is the probability that the particle has **not reached a at any time between 0 and t** .

To calculate the survival probability by integrating a spatial probability density, we must use a density that excludes particles as soon as they reach the target for the first time. Otherwise, trajectories that have already reached the target may still contribute to the integral and incorrectly be counted as survivors.

For example, consider unrestricted Brownian diffusion. The probability density $p_{\text{free}}(x, t)$ describes where the particle is at time t , regardless of whether it has previously reached the target. Therefore,

$$S_a(t) \neq \int_{-\infty}^a p_{\text{free}}(x, t) dx.$$

The integral also counts trajectories that reached a at some earlier time and subsequently returned to $x < a$. These trajectories have already undergone their **first passage** and must not be counted as survivors.

To exclude them, we solve the diffusion equation for a density $\rho(x, t)$ of particles that have not yet reached the target, with an **absorbing boundary condition**

$$\rho(a, t) = 0.$$

This means that each trajectory is removed from the surviving ensemble as soon as it reaches a for the first time. Consequently,

$$S_a(t) = \int_{-\infty}^a \rho(x, t) dx.$$

The absorbing boundary is a mathematical way to distinguish trajectories that have never reached the target from those that have already reached it and returned. It does not require a physical absorber.